Natural resource federalism: Preferences versus connectivity for patchy resources

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Abstract

We examine the efficiency of centralized versus decentralized management of spatially-connected renewable resources when users have heterogeneous preferences for conservation vs. extraction. Resource mobility and heterogeneity induce a spatial externality, while spatial preference heterogeneity drives a wedge between users' privately optimal extraction rates. We first address these market failures analytically and show that the first is most efficiently handled with centralized planning while the second is best tackled with decentralized management. Except in special cases, neither approach will be first best, but which arises as second best depends on the relative strength of preference heterogeneity versus spatial mobility of the resource. We illustrate the theory, and test its robustness, with a numerical example.

Keywords: renewable resources, federalism, spatial externalities, property rights

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1 Introduction & Background

We seek to determine the conditions under which spatially-connected renewable resources are more efficiently managed by a central planner or by decentralized property right holders. A rich and enlightening literature reveals that spatial concerns, such as mobility and heterogeneity in production, can significantly alter efficient management of renewable natural resources. Indeed, both the natural science and economics literatures have focused on characterizing these spatial issues and deriving efficient policy responses to them (Brown and Roughgarden 1997; Hastings and Botsford 1999; Sanchirico and Wilen 2005; Costello and Polasky 2008). A key finding is that a central planner, when armed with perfect scientific information about the spatial characteristics of the resource, can perfectly design a system of spatial harvests (Kaffine and Costello 2011), taxes (Sanchirico and Wilen 2005), and/or natural areas (Sanchirico et al. 2006) to maximize welfare over space and time.

Yet a second, at-least-as-ubiquitous source of spatial heterogeneity exists in the preferences of resource users themselves, and this source has gone practically unnoticed by these literatures.[1] People, residing in different spatial locations, may have different preferences for resource extraction versus conservation. The stark lack of treatment of this second source of heterogeneity is surprising for two reasons. The first reason is practical: preference heterogeneity underpins many of the greatest debates of the day regarding use of public trust natural resources such as fisheries, forests, and wild game. For example, commercial extractors, recreational users, providers of commercial eco-tourism services,

[1] An exception is Arnason (2009) who accounts for preferences but implicitly assumes an aspatial world. Arnason (2009) shows that an Individual Tradable Quota (ITQ) system between commercial users, recreational users, and conservationists can yield an efficient allocation, assuming each group can internally resolve the free-rider problem.

and conservationists will likely have very different notions of what constitutes optimal management of natural resources. The second reason is more academic: preference heterogeneity features prominently in the public economics literature on fiscal federalism and policy design (Oates 1999; Besley and Coate 2003; Alm and Banzhaf 2012),[2] in which a central issue is the optimal "scale" of policy - should decisions be made at the federal, state, local, or even individual level? Despite the obvious parallels to issues of spatial resource management, this literature has not bridged to natural resource economics, which introduces new challenges via intertemporal resource dynamics and spatial externalities. To address this gap, we formalize a theory of natural resource federalism, incorporating insights from the literature on both spatial natural resource economics and fiscal and environmental federalism.

To illustrate the renewable resource problems of interest, consider a typical coastal fishery in the developing tropics where different communities extract from a shared fish stock. Because fish move, one community's harvest imposes an externality on others. Often these communities will have different preferences; one may favor commercial extraction, another desires trophy size fish for recreational fishing, still another desires high biodiversity for scuba diving tourists, while yet another may prefer conservation for its own sake. Examples of these heterogeneous fisheries, where users differ in their preferences for conservation versus extraction, are not hard to come across and include iconic places such as Galapagos, Indonesia, and Baja California, Mexico. But the basic economic theory applies equally well for migratory game species such as lions and zebra in

[2] For example, Oates (1999) states: "By tailoring outputs of such goods and services to the particular preferences and circumstances of their constituencies, decentralized provision increases economic welfare above that which results from the more uniform levels of such services that are likely under national provision."

Africa, elk and wolves in the Yellowstone ecosystem, and waterfowl in the great migratory flyways of North America. In addition to the “traditional" challenges of managing a mobile renewable resource like those described above, resource managers may also have incomplete information regarding the preferences of the different communities.

While the established literature provides some loose guidance for solving the renewable resource management problems described above, to our knowledge none simultaneously address the spatial and dynamic aspects of renewable resource management with uncertainty and multiple sources of heterogeneity (preferences, growth, connectivity, economic productivity) across multiple users. Conceptually, List and Mason (2001) is the most closely related as it examines CO_2 emissions and compares a decentralized, asymmetric two-player game-theoretic outcome against a central planner that picks a one-size-fits-all policy. While dynamics emerge via the stock of uniformly mixed CO_2, this is a different class of dynamics relative to the intertemporal growth dynamics in renewable resources. Eichner and Runkel (2012) also shares some similarities in that they examine transboundary emissions in a setting where the capital stock in multiple symmetric districts can be affected by local tax policies via savings decisions. Growth of the capital stock is endogenized via a two-period model, however preferences are known with certainty and thus they compare the decentralized outcome with that of a perfectly informed social planner.

In this paper, we analytically compare alternative management regimes over a natural resource that exhibits spatial heterogeneity in both resource characteristics and preferences. Our “patchy” spatial environment allows for heterogeneity in resource productivity, connectivity, economic returns, and user preferences across the natural resource. We begin by deriving the first-best management of spatial resources given resource and preference heterogeneity as a benchmark. Simultaneously accounting for both resource production externalities (due to resource mobility) and spatial preference heterogeneity to maximize social welfare is an onerous task. A benevolent social planner must do so by accounting for the effect of harvesting in one location on the future stock in all other locations, and for the preferences in each location for extraction versus conservation. However, that precise level of spatial and temporal information and control is often unrealistic. Rather we will assume that the regulator has complete information about the spatial ecosystem dynamics, but incomplete information about the heterogeneity in preferences.

Under the assumption that the regulator only knows the *distribution* of preference types in the economy, we explore two second-best alternatives to the omniscient social planner. First, under the top-down approach of *Centralized Planning* (CP), the planner could utilize the ecosystem information to design spatial policy despite incomplete information about preferences.[3] Second, the planner could devolve all authority to decentralized users who know their own preferences, a bottom-up approach we denote *Decentralized Management* (DM). For example, the planner could assign spatial property rights and then the owners of those spatial property rights would select privately optimal extraction rates in their own areas and thus compete in a dynamic and spatial game against one another.[4] Returning to the examples discussed above, in each case a resource manager is charged with regulating extraction of a mobile natural resource and faces a fundamental challenge of whether to engage in *top-down* control, where she tries to set

[3]Such an approach is consistent with many real-world spatial natural resource settings. For example, in managed fisheries, it is typically the regulator, not the individual harvesters, who determines annual quotas.

[4]For the purposes of exposition, our concept of decentralized management is at the individual resource user, while centralized planning is at some higher level of authority (e.g. state). However, the key insights of our model can be applied whenever there are potentially different levels of natural resource management (state versus local, national versus state, or international versus national, etc.).

spatial regulations to satisfy the average preferences of her constituency, or to delegate *bottom-up* control, where local communities set local policies to manage their resource.[5]

Contrasting these two second best alternatives with the first best solution reveals an important tension in managing natural resources characterized by both spatial resource and preference heterogeneity.[6] On one hand, spatial management rules need to reflect heterogeneous spatial externalities arising from connectivity between resource patches. This has been the focus of nearly all of the spatial resource economics literature to date. But on the other hand, spatial management rules must also account for differences in preferences over how management is carried out over space. It then follows that while *centralized planning* may adequately capture spatial externalities between resources, only the average user is truly satisfied with the management rule due to lack of the information by the central planner regarding local preferences (Hayek 1945; Oates 1999). By contrast, *decentralized management* allows users to select private management rules reflecting precisely their specific preferences within each location, but will ignore any spatial externalities created across locations (Bhat and Huffaker 2007; Janmaat 2005). A concrete policy

[5]For example, consider the challenge of determining how many elk hunting permits to issue in each of 163 hunting districts in the state of Montana, recognizing that elk migrate and that preferences vary widely across districts from extreme conservationists (who would favor no hunting at all) to avid sportsmen (who might favor managing to population to maximize hunting opportunities).

[6]We note that our analytical treatment of the alternative institutional options is deliberately stylized to focus on the tension between resource and preference heterogeneity and derive the conditions under which CP or DM delivers the greatest social welfare. Nevertheless, there are clearly other factors that may influence why centralization or decentralization may be preferred for natural resource management. For example, the public choice literature raises important concerns about the incentives of centralized bureaucracies and regulators of natural resources (Anderson and Leal 1991). Another strand of literature emphasizes the potential for decentralized cooperation and coordination amongst resource users (Ostrom 1990). Our analysis should be viewed as complementary to these established literatures.

implication of our results is that if the primary challenge facing natural resource management is resource heterogeneity (and the resulting spatial externalities), then *centralized planning* may dominate *decentralized management.* However, if the primary challenge is differences in preferences of various users, then delegation under *decentralized management* may be the second-best management option. The analysis also reveals that the most socially challenging resources to manage are those that exhibit high degrees of both resource heterogeneity and preference heterogeneity. For that class of resources, neither the CP nor the DM approach will perform well, suggesting a high value from coordination to approach the first best solution. These intermediate cases are analyzed in greater detail with a numerical example.

2 A patchy resource model with spatial heterogeneity

Our discrete-time, discrete-space model extends Reed (1979) and closely follows Costello and Polasky (2008) and Costello et al. (2015). We describe the biological model, economic model, and governance structures below. We note that the various stylized assumptions of the biological and economic model below can be relaxed and explored numerically (see Section 5.2). Nonetheless the particular structure we adopt allows us to generally capture the features (e.g heterogeneity in growth, connectivity, economic returns, and preferences) of the research question questions we examine while maintaining analytical tractability.

2.1 Growth and movement

We assume that the resource consists of N spatially-connected patches (a "metapopulation" model). The timing is such that the resource stock in patch i ($i = 1, ..., N$) at the beginning of period t is given by x_{it}. This stock is then harvested h_{it}, yielding the residual stock (i.e. *escapement*) at the end of the period given by $e_{it} \equiv x_{it} - h_{it}$. The residual stock grows and then disperses, whereby concave growth in patch i is denoted $f_i(e_i)$, which may be patch specific, and the fraction of the stock that moves from patch j to i is given by D_{ji} (where D is an N x N matrix of dispersal).[7] Thus, the resource dynamics in patch i are given by:

$$x_{it+1} = \sum_{j=1}^{N} f_j(e_{jt}) D_{ji}. \tag{1}$$

Resource growth may differ across space (indicated by the subscript on $f_j(e_{jt})$), which may be driven by differences in ecology and habitat, while heterogeneity in the dispersal relationships D_{ji} may be driven by differences in oceanography, wind patterns, trade volumes, or other processes. We assume that for each of these resource patches, there is a single agent who exclusively manages her own patch in discrete time periods, $t = 0, 1, 2, ...$, either by selecting an escapement level herself (under DM) or by executing an escapement plan specified by a central authority (under CP).[8] As noted in the previous literature (e.g.

[7] Consistent with the previous literature, the growth function satisfies the conditions: $f_i'(e) > 0, f_i''(e) < 0$ and $f_i(0) = 0$. Note that specifying different timing, such as dispersal occurring before growth, or alternative representations of dispersal, for example density-dependent dispersal, would affect analytical tractability. See Costello and Polasky (2008) for further details.

[8] Of course, this individual agent could represent more than one person. For example, if resource patches represent a series of Territorial Use Right Fisheries (TURFs), then the agent's decisions may represent the collective will of the users of that particular TURF (Cancino et al. 2007; Wilen et al. 2012).

Reed (1979), Kapaun and Quaas (2013), and Costello et al. (2015)), choosing residual stock e_{it} or harvest h_{it} as the decision variable is formally equivalent; however, selecting residual stock as the decision variable is mathematically more convenient.

2.2 Economic model

In addition to heterogeneity in growth and movement, we also allow for heterogeneity in economic returns and preferences. Preferences reflect the utility derived from economic returns versus residual stock, for example from recreational or conservation benefits. Patch-i sub-utility in period t associated with economic returns on harvest h_{it} is given by $(p_i(\underbrace{x_{it} - e_{it}}_{\equiv h_{it}}))^{\beta_h}$, where $p_i > 0$ is the net price per unit harvest in patch i and $\beta_h \leq 1$ allows for potential decreasing returns in economic outcomes.[9] The period-t sub-utility in patch i derived from the residual stock is given by $(ke_{it})^{\beta_e}$ where the parameter k reflects the marginal benefit of residual stock and $\beta_e \leq 1$ allows for potential decreasing returns to residual stock.[10] The preference for extraction profit relative to resource stock is given by the patch-specific parameter, α_i. Thus, total period-t utility in patch i is given by:

$$U_{it}(x_{it}, e_{it}) = \alpha_i(p_i(x_{it} - e_{it}))^{\beta_h} + (1 - \alpha_i)(ke_{it})^{\beta_e} \tag{2}$$

where $\alpha_i \in (0, 1)\ \forall i$. Greater preference heterogeneity is represented by a greater range across α_i; preferences are identical if and only if $\alpha_i = \alpha_j, \forall i, j$.

The parameter α_i is meant to capture a broad range of different user preferences for natural resources. For example, if α_j is near 1, then patch j places weight primarily on

[9]Net price may vary across patches due to spatial heterogeneity in the underlying resource quality or due to the patch-specific cost of harvest.

[10]We initially assume that preferences and thus utility are defined over local (patch-specific) residual stock e_{it}. We also consider the case where preferences are defined over global residual stock $\sum_j e_{jt}$ in an extension below.

extraction profits (a pure harvester - the case almost exclusively analyzed by the resource economics literature), while α_j near 0 represents a conservationist who gains little utility from extraction relative to maintaining a large resource stock (a pure conservationist).[11] Intermediate levels of α_j could represent recreational users who may derive benefits from some use or extraction, but also value an increasing resource stock.[12] As a heuristic, consider a coastal fishery exploited by three communities. Village A favors extraction, village B caters to recreational trophy-seeking clients, and village C caters to scuba diving tourists. In that case $\alpha_A > \alpha_B > \alpha_C$. Another example concerns waterfowl that move between area Y (a wildlife refuge with bird-watchers) and area Z (a hunting club). There $\alpha_Y < \alpha_Z$.

2.3 Governance Regimes

Our main objective is to analyze welfare and resource outcomes of this model under three distinct governance regimes to highlight tradeoffs between centralized planning and decentralized management for this class of problems. We initially consider the first-best (FB) solution to the above problem, where a fully-informed social planner determines optimal spatial extraction in each time period and each of N resource patches to maximize the present value sum of utility over all agents. To the best of our knowledge this benchmark model has never been proposed or solved. We solve this problem analytically

[11] Note that by conservation, we mean conservation for its own sake. In our dynamic context, users who place weight on extraction also have some incentive for conservation to the extent that it increases the discounted stream of future extraction profits.

[12] For example, recreational fishermen may place positive weight on resource stock size to the extent that a larger stock translates to a larger fish size (as is the case in a delay-difference model (Hilborn and Walters 1999)), increasing the probability of catching a trophy fish.

and use the solution as a benchmark against which to compare other management regimes.

While the FB approach will (in the absence of information or management costs) deliver an ideal aggregate welfare result, obtaining the necessary information regarding preferences in each resource patch may be costly or infeasible. As such, we consider the second best efficiency of alternative management regimes given asymmetric information about preferences. Will a higher level of utility be achieved by centrally planning harvest in each patch (without knowing each agent's preferences), or by devolving management decisions (e.g assigning spatial property rights) to individual patch owners who each know their own preferences but compete non-cooperatively over extraction? Under this model, global utility from FB can never be exceeded by either of the other approaches. Thus our main goal is to rank regimes, CP versus DM, under different assumptions about the underlying resource (e.g. how heterogeneous is resource production and connectivity across space) and the underlying preferences (how heterogeneous are preferences across space). But along the way we will be able to explicitly solve the dynamic spatial optimization problems under each of the three governance regimes.

3 First-best solution

We begin with a purely theoretical analysis, where we adopt two assumptions for tractability. First, we assume period-t utility in patch i is linear in economic returns and extant resource stock:

Assumption 1. $\beta_h = \beta_e = 1$.

Second, we assume parameter values and growth functions are such that optimal escapement choices are interior for all governance regimes:

Assumption 2. *The parameters* $\{\alpha_i, D, k, p_i\}$ *and growth function* $f_i(e_i)$ *are such that an interior solution exists* $(0 < e_{it} < x_{it},\ \forall i)$ *across all governance regimes.*

The above assumptions will allow us to obtain sharp analytical solutions below, while in the numerical section we explore relaxations of these assumptions (non-linear utility, the presence of corner solutions).[13]

The first-best solution to the above spatial dynamic problem consists of a harvest plan chosen by a social planner who can synchronize harvest decisions across space and time to maximize the present value of global utility. This omniscient planner must simultaneously account for the heterogeneous resource dynamics and heterogeneous preferences. Letting $\mathbf{x_t}$ represent the vector of stocks $[x_{1t}, ..., x_{Nt}]$ and $\mathbf{e_t}$ represent the vector of residual stocks $[e_{1t}, ..., e_{Nt}]$, the dynamic programming equation for the first-best problem is thus:

$$V_t(\mathbf{x_t}) = \max_{\mathbf{e_t}} \sum_{i=1}^{N} (\alpha_i p_i (x_{it} - e_{it}) + (1 - \alpha_i) k e_{it}) + \delta V_{t+1}(\mathbf{x_{t+1}}) \tag{3}$$

If any resource or preference heterogeneity exists (e.g. if $f_i(e_i) \neq f_j(e_j)$, $D_{ij} \neq D_{kl}$, or $\alpha_i \neq \alpha_j$), then different harvest policies across space will be chosen. This is a complex problem, but different versions (which lack conservation utility and preference heterogeneity) have been addressed previously by Costello and Polasky (2008) and Kaffine and Costello (2011).

Differentiating with respect to residual stock e_{it} gives the following necessary condition for an interior solution:

$$-\alpha_i p_i + (1 - \alpha_i) k + \delta \sum_{j=1}^{N} \frac{\partial V_{t+1}(\mathbf{x_{t+1}})}{\partial x_{jt+1}} \frac{\partial x_{jt+1}}{\partial e_{it}} = 0 \quad \forall i \tag{4}$$

The first term in Equation 4 represents the present period marginal costs of foregone harvest, the second term represents the present period marginal conservation benefits

[13] In the numerical exercise, we also confirm that there exist parameter values that do in fact generate interior solutions $\forall i$ across all governance regimes per Assumption 2.

of additional residual stock, and the third term captures the marginal benefit in future periods to all patches from residual stock growth and dispersal from patch i.

Our first result is to show that Equation 4 has a closed form solution, summarized below:

Proposition 1. *First-best residual stock in patch i is time-independent and is given by:*

$$f_i'(e_i^{FB}) = \frac{1}{\delta}\left[\frac{\alpha_i p_i - k(1-\alpha_i)}{\sum_{j=1}^{N}\alpha_j p_j D_{ij}}\right] \tag{5}$$

Proof. All proofs are provided in the Appendix. □

Proposition 1 represents a first-best, spatial golden rule for a spatially-connected renewable resource with heterogenous resource characteristics and user preferences. The FB decision maker should extract the resource in patch i down to the level indicated by Equation 5. Doing so in every patch and every period in perpetuity ensures the maximal present value of utility across the entire spatial domain.[14] Because $f_i(e)$ is an increasing concave function, it is straightforward to show that this policy implies maintaining a lower residual stock level in patch i when (1) the discount factor is low (i.e. the future is heavily discounted), (2) the marginal conservation value (k) is low, and/or (3) the patch-specific price p_i is high. The denominator captures the complicated link between preferences, economic returns, and connectivity between patches. Despite this complexity, it is clear that residual stock in patch i will be higher if it primarily disperses to relatively valuable patches (p_j is high).

[14]Crucially, the fact that the optimal choice of residual stock is independent of the state variable is a result and not an assumption. Furthermore, it has nothing to do with our choice of escapement as the control. The exact same result would obtain had we used harvest as the control. The FB decision maker is accounting for all spatial and dynamic consequences in each and every time period when choosing the residual stock level above, as the residual stock in patch i in period t does effect x_{jt+1} in all connected patches.

The following corollary summarizes the relationship between first-best residual stock and connectivity and preferences.

Corollary 1. *The first-best residual stock in patch i is increasing in self-retention (D_{ii}), out-dispersal (D_{ij}), i's utility preference for conservation ($1-\alpha_i$), and utility preference for extraction in other connected patches (α_j).*

While Equation 5 ensures first best management, calculating and implementing this optimal spatial extraction pattern would require detailed spatial information that may be difficult in practice to obtain. In the next section, we consider two second-best policies: a centralized policy that makes use of information on the distribution of preferences, versus a decentralized policy that delegates decision-making to individual patch owners.

4 Centralized Planning vs. Decentralized Management

4.1 Centralized planning

Under centralized planning (CP), a well-meaning central planner is completely informed about the underlying resource dynamics but knows only the distribution from which preferences are drawn, not the individual preferences in each patch location.[15] We first derive a useful lemma.

[15] This may be a generous assumption for how central resource managers actually behave. Indeed, they may attempt to accommodate users with different preferences to some extent, but whether they also explicitly account for the underlying spatial resource heterogeneity is the subject of much debate in the literature (see Sanchirico and Wilen (2005)). This suggests a possible third institutional regime in which the central planner ignores both resource heterogeneity *and* preference heterogeneity.

Lemma 1. *The expected utility under a distribution of α is equal to the utility under the expected preference parameter, $\bar{\alpha}$.*

Lemma 1 conveniently establishes that, due to the linearity of utility, the value function and necessary conditions under CP can be determined by replacing α_i with $\bar{\alpha} \equiv \sum \alpha_i/N$ in Equations 3 and 4. Under Centralized Planning the optimal harvest policy in patch i is summarized in the following proposition:

Proposition 2. *Under Centralized Planning, optimal residual stock in patch i is time-independent and is given by:*

$$f_i'(e_i^{CP}) = \frac{1}{\delta}\left[\frac{\bar{\alpha}p_i - k(1-\bar{\alpha})}{\bar{\alpha}\sum_{j=1}^{N} p_j D_{ij}}\right] \tag{6}$$

Proposition 2 establishes that, like the FB, the CP also has time-independent, but patch-dependent, residual stocks. Importantly, while Proposition 2 reveals that the optimal residual stock for the Centralized Planner in patch i will depend only on the average preferences, the residual stock still differs across space. For example, compare two patches A and B that are identical except for their dispersal characteristics: A tends to disperse resource stock toward high value patches (those with high prices) while B tends to disperse toward low value patches. Inspecting Equation 6, patch A will have a large denominator on the right hand side such that the optimal residual stock will be larger in A than in B. This accords with economic intuition but ignores possible differences in preferences across space. To streamline the analysis, we will occasionally make use of a condition that renders patches symmetric with respect to some characteristics,[16] as follows:

[16]Note, however, that even under Condition 1 patches can differ in resource production ($f_i(\cdot) \neq f_j(\cdot)$) and preferences ($\alpha_i \neq \alpha_j$), and out-dispersal from a patch can differ from self-retention within that patch ($D_{ik} \neq D_{kk}$).

Condition 1. $p_i = p\ \forall i$, $D_{ij} = Q\ \forall j \neq i$, *and* $D_{ii} = D\ \forall i$.

While we will not require Condition 1 for most of our results, it will in some cases enable us to prove necessity of certain results (when without it, we could only prove sufficiency). We will be explicit about when Condition 1 is being invoked. We will also occasionally make use of the additional condition on the growth functions:

Condition 2. $f_i(e) = f(e)\ \forall i$ *and* $f'''(e) \geq 0$.

The following corollary compares the Centralized Planner's policy to the first-best policy.

Corollary 2. *a. If* $\alpha_i = \bar{\alpha}\ \forall i$, *then* $e_i^{CP} = e_i^{FB}$.

b. Under Condition 1, if $\alpha_i \binom{<}{>} \bar{\alpha}$, *then* $e_i^{CP} \binom{<}{>} e_i^{FB}$.

c. Under Condition 1, if $e_i^{CP} = e_i^{FB}\ \forall i$, *then* $\alpha_i = \bar{\alpha}\ \forall i$.

d. Under Conditions 1 and 2, global residual stock is larger under FB than under CP.

Thus, under homogeneous preferences, CP exactly replicates the first-best solution. But when preferences are heterogeneous the harvest rules diverge. There are two reasons for this. Inspecting the numerator of Equation 6, consider a patch i for which preferences lean toward conservation ($\alpha_i < \bar{\alpha}$). In that setting, the central planner would call for excessively high extraction in patch i (the reverse is true if $\alpha_i > \bar{\alpha}$). Inspecting the denominator, centralized planning ignores the fact that connectivity results in dispersal to patches and may leave too little or too much residual stock relative to FB.[17] Under Conditions 1 and 2 it is also possible to show that the global stock (summed over the entire spatial domain) is larger under FB than under CP.

[17] In other words, centralized planning is equivalent to assuming that the preference for additional extraction revenue from dispersed stock is identical regardless of where that dispersal occurs.

4.2 Decentralized Management

An alternative institutional arrangement is Decentralized Management (DM) under which there is no coordinating central planner. Rather, spatial property rights are defined over each of the N resource patches and each is managed by a single agent who optimizes the harvest decisions in his own patch to maximize his own utility, conditional upon the choices made in all other patches. Thus, the N private property right holders interact in a spatial dynamic game.

Solving for the optimal feedback control rule for owner i and finding the equilibrium of these rules across N owners is a demanding task. But it turns out that this game has a special structure under which the optimal harvest strategy for owner i depends linearly on the state in patch i. This special structure implies that while the strategies will differ across patches, the equilibrium residual stock in patch i is time-independent and can be written as an explicit function of only patch i parameters. This result is summarized below.

Proposition 3. *Under Decentralized Management, the patch i optimal residual stock is time-independent and is given by:*

$$f_i'(e_i^{DM}) = \frac{1}{\delta}\left[\frac{\alpha_i p_i - k(1-\alpha_i)}{\alpha_i p_i D_{ii}}\right] \tag{7}$$

Intuitively, then, one might expect special cases to arise under which Decentralized Management gives rise to harvest policies (and thus welfare) that are identical to those under first-best management. Indeed, such cases exist, as is summarized below:

Corollary 3. *a.* $e_i^{DM} = e_i^{FB} \iff D_{ij} = 0, \forall j \neq i.$

b. $e_i^{DM} < e_i^{FB} \iff D_{ij} > 0$ *for some* $j \neq i$.

c. Global stock under FB exceeds global stock under DM $\iff D_{kl} > 0$ *for some* $k \neq l$.

Out-dispersal from patch i (D_{ij}) always reduces the residual stock, and as such, any externality from patch i drives a wedge between the harvest policies under DM and the FB.

4.3 Centralization versus decentralization

Our main objective is to derive the conditions under which society would prefer a centralized planner approach (despite incomplete information regarding preferences) versus a decentralized property rights approach (under which owners compete noncooperatively). The results above immediately reveal cases under which one or the other of these reproduces the first best, and is thus strictly preferred by society:

Proposition 4. *a. With preference heterogeneity, but no resource externality ($\alpha_i \neq \alpha_j$ for some i, j and $D_{ij} = 0\ \forall i \neq j$), DM exactly reproduces the first-best and (in general) CP does not, thus DM $\succ$ CP.*[18]

b. With a resource externality, but no preference heterogeneity ($\alpha_i = \alpha_j\ \forall i, j$ and $D_{ij} > 0$ for some $i \neq j$), CP exactly reproduces the first-best and DM does not, thus DM $\prec$ CP.

Proposition 4a sharpens a loose intuition that motivated this paper: decentralized management approaches, such as assigning spatial property rights, can perfectly solve the problem of spatial heterogeneity in preferences, while the centralized approach, with imperfect information over preferences, cannot. Thus, in the absence of spatial externalities, the decentralized approach is preferred to the centralized approach. By contrast,

[18] The qualifier "in general" refers to the fact that we earlier invoked Condition 1 to prove necessity. Even if Condition 1 fails to hold, the result still typically holds, but there are special parameter combinations where it would not hold.

when we exchange the source of the problem, so spatial externalities are present, but spatial heterogeneity in preferences is absent, this result is reversed. Under the conditions of Proposition 4b the institutional challenge is that the underlying resource itself produces an externality in the classical common-pool sense; each patch recognizes that some fraction of the resource produced on its patch will be captured by its neighbors. Thus, in this setting, Decentralized Management drives each owner to over-extract the resource. By contrast, Centralized Planning will completely internalize this externality, and in a case where all owners have identical preferences ($\alpha_i = \bar{\alpha} \ \forall i$), no problem arises when the central planner effectively averages across owners' preferences. Thus, in this case, the centralized approach is preferred to the decentralized approach.

While these special cases help sharpen intuition, they only give a loose sense of institutional design in the presence of both preference heterogeneity and spatial externalities. It turns out that the intuition derived above continues to hold outside of these special cases. We prove this mathematically with a continuity argument and we summarize the result as follows:

Proposition 5. *a. If the primary challenge facing resource managers is differences in preferences of various users, then $DM \succ CP$.*

b. If the primary challenge facing resource managers is resource connectivity, then $DM \prec CP$.

Proposition 5 reveals that even when both challenges are present, provided that one effect is sufficiently large and the other is sufficiently small, we can unambiguously sign society's preference of DM over CP (or vice versa). However, this sharp theoretical result is eroded when both sources of heterogeneity are present and significant. In those cases, neither DM nor CP is first best and which is second best becomes an empirical question.

For that class of problems, the solutions from Propositions 1-3 provide the necessary calculations to compare DM to CP; we will do so in Section 5.2.

5 Extensions, refinements, and an illustrative example

While the model setup introduced in Section 2 was quite general, obtaining analytical results required making some fairly restrictive assumptions. Furthermore, while our theoretical results in Propositions 1-5 provide general insights about the conditions under which decentralization will be welfare-enhancing, the model complexity prevented us from making concrete predictions about the behavioral and welfare effects in the presence of *both* resource and preference heterogeneity. While we have argued that our qualitative results are unlikely to be altered by reasonable relaxation of our assumptions, we will examine this conjecture with a numerical analysis. In this section we undertake a number of supplementary analyses designed to test the robustness of our results to model assumptions.

5.1 Global vs. local conservation preferences

We begin by examining a change in the way we model conservation preferences. In our original model, the agent in patch i derived conservation utility only from resource stock in patch i and did not explicitly derive utility from resource stock in other locations. We call these "local" conservation preferences. The assumption of local conservation preferences is probably appropriate for many *use* values such as scuba diving and eco-tourism, but may not be appropriate for many *non-use* values such as the existence of biodiversity.

Thus, an equally reasonable assumption is that agents derive conservation utility from the aggregate (system-wide) resource stock, regardless of where the agent physically resides. To account for this possibility, here we will allow conservation demand to depend on the aggregate stock, which (invoking Assumption 1) gives rise to patch i utility of:

$$U_{it}(x_{it}, \mathbf{e}_t) = \alpha_i p_i(x_{it} - e_{it}) + (1 - \alpha_i)k \sum_{j=1}^{N} e_{jt} \tag{8}$$

Following the above methodology, we derive optimal harvest rules under this change in assumptions. The resulting optimal residual stocks are summarized as follows:

Proposition 6. *When conservation preferences are defined over aggregate residual stock in the spatial system, the optimal residual stock under FB, CP, and DM are respectively given by:*

$$f_i'(\hat{e}_i^{FB}) = \frac{1}{\delta}\left[\frac{\alpha_i p_i - k\sum_{j=1}^{N}(1-\alpha_j)}{\sum_{j=1}^{N}\alpha_j p_j D_{ij}}\right] \tag{9}$$

$$f_i'(\hat{e}_i^{CP}) = \frac{1}{\delta}\left[\frac{\bar{\alpha} p_i - kN(1-\bar{\alpha})}{\bar{\alpha}\sum_{j=1}^{N} p_j D_{ij}}\right] \tag{10}$$

$$f_i'(\hat{e}_i^{DM}) = \frac{1}{\delta}\left[\frac{\alpha_i p_i - k(1-\alpha_i)}{\alpha_i p_i D_{ii}}\right] \tag{11}$$

A comparison between the above expressions and their counterparts (Equations 5, 6 and 7) reveals a number of useful insights. First, $\hat{e}_i^{FB} > e_i^{FB}$, so the efficient residual stock under global conservation preferences is always larger than that under local preferences. However, $\hat{e}_i^{DM} = e_i^{DM}$, so the decentralized property owner will not change her residual stock under global preferences. Since $e_i^{DM} < e_i^{FB}$ (see Corollary 3), this suggests that global preferences always exacerbate the distortion caused by decentralization. The effect of global conservation preferences on residual stock for CP are less clear. Here, $\hat{e}_i^{CP} > e_i^{CP}$, so (like FB) Centralized Planning will increase residual stock under global preferences. It turns out that under global preferences, the distortion from CP can either grow or shrink. These results are summarized as follows:

Corollary 4. *Compared to the case of local conservation preferences, global conservation preferences have the following effects:*

a. Residual stock under DM is farther from first-best

b. Residual stock under CP can either be farther, or closer, to first-best.

When conservationists have global preferences, the distortion caused by decentralized management is exacerbated, while the distortion caused by centralized planning can be abated. Corollary 4a is a direct consequence of decentralized property right owner behavior - global preferences do not affect individual property owner behavior (because they can only influence local behavior). Because decentralized managers ignore the effect of their residual stock decisions on global conservation preferences, an additional externality is created (in addition to the production externality associated with spatial connectivity across patches). Corollary 4b is more nuanced and claims that the gap between CP and FB residual stock can either grow (as it did under DM) or shrink (which would imply a smaller distortion). While the proof requires careful analysis, some intuition can be gleaned. First, suppose out-dispersal (D_{ij} for $j \neq i$) is small. In this case, the resource externality is small so even though CP will fail to capture heterogeneous preference, she does a reasonable job of replicating FB residual stocks. In that case, invoking global preferences turns out to exacerbate the distortion. Instead, if out-dispersal is large, then the spatial dynamics become even more important - CP not only fails to capture local preference heterogeneity, but also fails to capture the important effects of dispersal to patches with different preferences. In this case, the distortion shrinks under global preferences. These nuances are more carefully drawn out in the proof to Corollary 4.

We have shown that global conservation preferences always imply larger stocks under efficient spatial management (FB), that the distortion caused by decentralization always grows, and that the distortion caused by centralized planning can either grow or shrink depending on system characteristics. This analysis can have important policy and welfare implications. First, it suggests that even if decentralized management works "pretty well" under local conservation preferences, it may fail miserably under global conservation preferences. Conversely, even if centralized planning was far from efficient under local conservation preferences, it may perform quite well under global conservation preferences. Finally, in cases in which the distortion is exacerbated under global conservation preferences (which is a large class of cases), the analysis suggests an even greater importance of moving toward first-best management of the spatial system.

5.2 Numerical example

Returning to the base case of local conservation preferences, we now develop an illustrative numerical example to test the robustness of results to several model assumptions. In particular, we undertake three numerical experiments. First, we use the numerical example to address the welfare preference for DM versus CP in the presence of *both* preference and resource heterogeneity. This involves numerically solving the first-order conditions (Equations 5, 6, and 7), and simulating the resulting present value utility under each regime. Second, we explore the possibility of corner solutions. Our analytical results are proven only when residual stocks are "interior" (i.e. under Assumption 2). Alternatively, corners may be possible in which a given patch owner finds it optimal to extract *all* of the resource stock (so $e_{it} = 0$) or *none* of the resource stock (so $e_{it} = x_{it}$). Examining how corner solutions affect our main theoretical results involves the complex task of numerically solving the spatial-dynamic optimization problem and game that arises. The high-dimensionality of the state and control space make this a challenging problem to solve numerically. The third numerical experiment involves asking whether the presence of nonlinearities in utility will qualitatively affect our conclusions. In particular, we allow the utility function to be nonlinear in harvest and/or resource abundance. Again, this involves numerically solving a complicated spatial-dynamic optimization and game.

To accomplish these tasks, we develop an illustrative three-patch model with the following features and parameterization:

- Resource growth in patch i is given by: $f_i(e) = e + r_i e(1 - e/K_i)$, where $r = [.523, .527, .438]$ and $K = [130.3, 121.8, 102.6]$.

- Utility is given by: $U_i(e_i, x_i) = \alpha_i (x_i - e_i)^\beta + (1 - \alpha_i)(k e_i)^\beta$, where α_i is given below and we examine a range of values for $k > 0$ and $\beta \leq 1$.

- Resource movement (dispersal) is given by $D =$

$$\begin{pmatrix} .813 & Q & Q \\ Q & .771 & Q \\ Q & Q & .718 \end{pmatrix},$$

  where we examine a range of values for the parameter Q.

- Environmental preferences are given by: $\alpha = [.5 - \varepsilon, .5, .5 + \varepsilon]$, where we examine a range of values for the parameter ε.

While this parameterization is meant to be illustrative only, it builds on the numerical example provided in Costello et al. (2015), where we aggregate patches by island. Under this parameterization, increased preference heterogeneity is controlled by ε and increased resource heterogeneity is controlled by Q.

5.2.1 Federalism under resource *and* preference heterogeneity

One of the key findings of our theoretical model is that decentralizing management of natural resources is first-best in the absence of resource movement and central planning is first-best in the absence of preference heterogeneity (Proposition 4). We also showed that, while neither will be first best if *both* types of heterogeneity are present, as long as resource movement is "small," DM will still dominate (and as long as preference heterogeneity is "small," CP will still dominate). But if both types of heterogeneity are present and are sufficiently large, our analytical results thus far provide little guidance about management regime choice. To address this issue, we employ Equations 6 and 7, which are the first-order conditions (given an interior solution exists) defining the equilibrium optimal residual stocks under central planning and decentralized management, respectively. To explore the welfare preference of CP vs. DM as a function of the degree of preference heterogeneity (parameterized by ε above) and resource heterogeneity (parameterized by Q above), we solve Equations 6 and 7 over a large parameter space of ε and Q (Figure 1).

Figure 1 displays a parameter space of $\varepsilon \in [0, .16]$ and $Q \in [0, .1]$. All solutions in this parameter space are interior (that is, they satisfy Assumption 2) under all regimes, so Equations 5, 6, and 7 apply exactly. For any given combination of parameters within this space, we calculate the optimal residual stocks across the three patches and the resulting system-wide present value of utility. The curve in Figure 1 divides the parameter space into two regions. Above the line, utility under CP exceeds utility under DM, and below the line the opposite result holds. We indicate welfare loss relative to FB by the shading in Figure 1, where darker shading indicates greater loss. Several interesting findings derive from this figure. First, consistent with Proposition 4, there is no loss from implementing

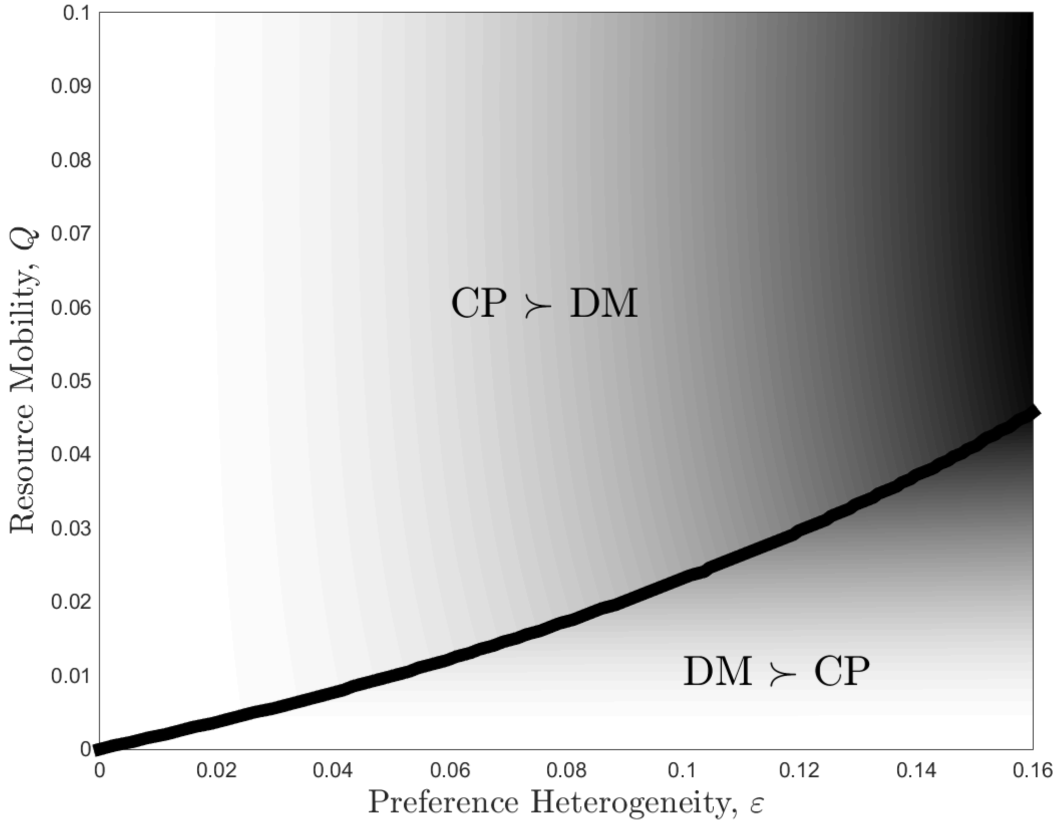

Figure 1: Parameter space over which central planner or decentralized management dominates. Darker shading indicates greater welfare loss under the preferred second best regime relative to first best.

CP (relative to FB) provided preference heterogeneity is zero, and there is no loss from implementing DM provided resource mobility is zero (shading is white along the axes). Second, consistent with Proposition 5, if preference heterogeneity is "small," CP clearly dominates DM, and if resource mobility is "small," DM clearly dominates CP. Third, the figures makes it clear that the loss from second-best management (whether CP or DM) is largest when both kinds of heterogeneity are large (shading is darkest as you move up and right on the figure). Finally, the curve in Figure 1 provides a precise numerical illustration of the conditions under which CP dominates DM, and vice-versa.

Figure 2 displays the results from three different parameterizations, whereby either resource or preference heterogeneity is fixed. The first row of Figure 2 corresponds to the parameterization described above; the bottom two rows are discussed below. The vertical axis of all subplots shows the present value utility of DM (relative to FB, solid line) and CP (relative to FB, dashed line), and the horizontal axes display variation in either resource (left panels) or preference heterogeneity (right panels). Focusing on the top row panels of Figure 2, when $Q = 0$ (so there is no resource mobility), DM achieves the same level of utility as does FB. However, increasing resource mobility Q erodes welfare under DM and eventually CP is preferred (left subplot). The right subplot shows that when $\varepsilon = 0$ (so there is no preference heterogeneity), CP achieves the same utility as FB.

5.2.2 Corner solutions

The analytical results from this paper assume interior solutions. While this is a common assumption in resource models (and is likely to be empirically relevant in many real-world cases) there are also interesting cases in which we might expect this assumption to be violated. Two kinds of corner solutions are possible. First, a patch owner may find it optimal to extract the entire resource stock from her patch, so $e_{it} = 0$. This might occur, for example, if her self-retention is small and she does not place much weight on conservation. The second kind of corner occurs when a patch owner (or social planner) wishes to leave a residual stock that exceeds the starting stock. Hitting this corner implies $e_{it} = x_{it}$. Either case presents a technical challenge as the first order conditions provided in Equations 5, 6, and/or 7 would no longer apply. A natural question is: Suppose the parameters are such that a corner solution obtains, what will be the effect on our main analytical results?

To examine this question we numerically solved the dynamic spatial optimization problem (for FB and CP) and the dynamic spatial game (for DM) using the parameterization described above. In the case of FB and CP, the state of the system has three dimensions (x_{1t}, x_{2t}, and x_{3t}), and the control has three dimensions (e_{1t}, e_{2t}, and e_{3t}). Aside from the growth and dispersal constraints, we also have $0 \leq e_{it} \leq x_{it}$. We solved the optimization problem for FB and CP using numerical dynamic programming techniques (backward induction using value function iteration). Solving the DM problem involved the additional step of calculating the best response functions for each of the three owners, at each time step for each possible state, and finding the fixed point of those best response functions.

For some parameters (e.g. those in Figure 1 and those in the top panels of Figure 2), interior solutions exist under all regimes (FB, CP, DM); in those cases, our numerical backward-induction results match the results from simply solving the analytical first order conditions. But for other parameters, corner solutions exist in some patches, for some regimes. The purpose of this subsection is to determine whether our main result, that CP is first-best without preference heterogeneity and DM is first-best without resource mobility, still holds under corner solutions. This analysis is displayed in the middle row panels of Figure 2; all parameters represented in those panels result in corner solutions. In the absence of resource mobility (so $Q = 0$, left panel), profit under DM is equivalent to profit under FB, but profit under CP is not; this supports Proposition 4a. In the absence of preference heterogeneity ($\varepsilon = 0$, right panel), CP reproduces FB, but CP does not; this supports Proposition 4b.

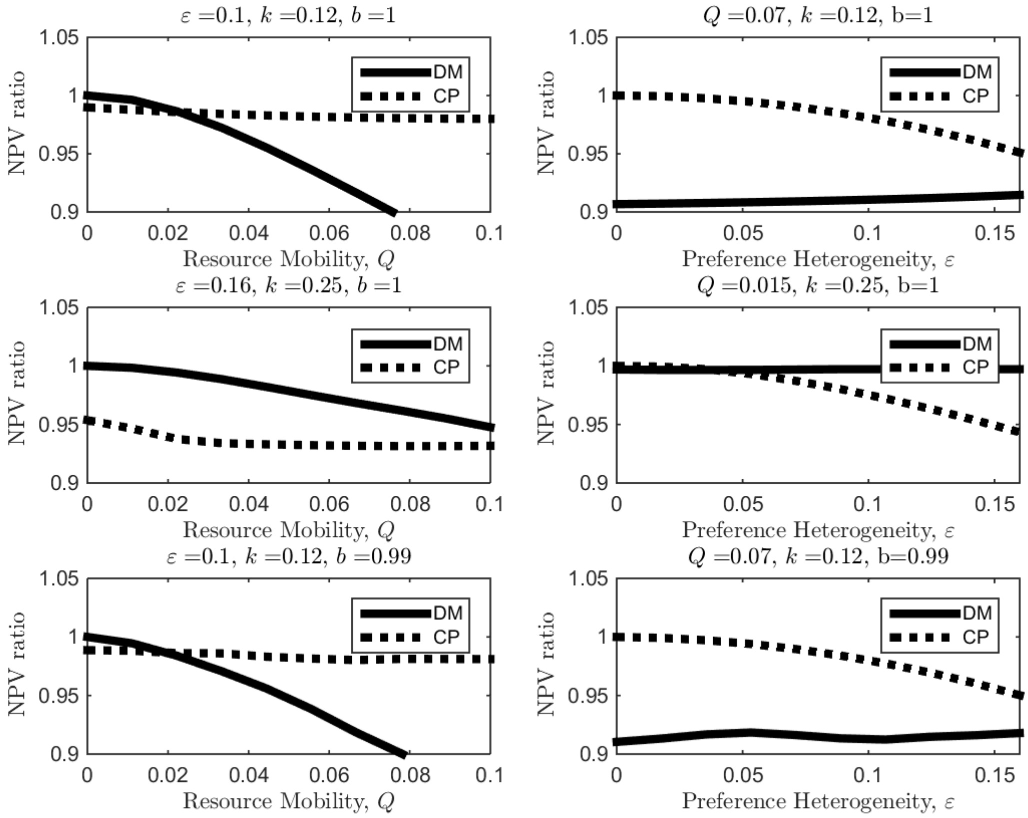

Figure 2: Welfare of DM versus welfare of CP (both relative to FB) for different parameter values. Top panels illustrate interior solutions for the numerical model in Section 5.2. Middle panels illustrate corner solutions and bottom panels illustrate solutions for the nonlinear utility model.

5.2.3 Model nonlinearities

Deriving analytical results required making some special assumptions about the utility function. Namely, our analytical results rely on Assumption 1, whereby extractive utility is linear in harvest and conservation utility is linear in residual stock. While these assumptions may make sense in some systems, it is easy to think of real-world exceptions that would violate this assumption. For example, if demand facing a fishery is downward-sloping, then utility would be a concave function of harvest. Concave utility establishes a link between time periods, and would destroy the *time independent* nature of optimal residual stock in each patch. While this would substantially complicate the analytics because it fundamentally changes the strategies pursued in each patch, we do not anticipate that introducing small nonlinearities will alter the qualitative conclusions of this model (referring to Proposition 4). The purpose of this section is to test that claim using the numerical model described above.

The numerical dynamic-spatial model discussed above is agnostic about the degree of nonlinearity on utility (parameterized by $\beta = \beta_h = \beta_e$), so it is a straightforward matter of examining how $\beta < 1$ affects Proposition 4. The bottom rows panels of Figure 2 analyze this case.[19] Two main results are worth noting. First, even though introducing nonlinear utility alters the residual stock outcomes, Proposition 4 seems to stand - DM is still first-best when $Q = 0$, and CP is still first-best when $\varepsilon = 0$. Second, comparing the top row of panels in Figure 2 with the bottom row of panels in Figure 2, incorporating small nonlinearities in utility have only a small impact on overall utility and on the ranking across policies. This numerical evidence seems to support our conjecture that, while nonlinear utility does substantially complicate the analytics, incorporating small nonlinearities in utility does not qualitatively affect the main findings in this paper.

5.3 Further discussion

A number of other modeling assumptions deserve further discussion. First, we have assumed that costs are proportional to harvest. Instead, for some resources, marginal harvest costs may vary inversely with the resource stock. As the stock is drawn down,

[19]Except for β_h and β_l, the parameters in the bottom panels of Figure 2 match the parameters from the top panels.

the resource becomes less dense, and more costly to extract (Clark 1990). While we have not explicitly analyzed that case here, we speculate that stock effects may push CP to be more preferred - a CP will account for how residual stock (and dispersal) from patch i affects marginal harvest costs in patch j, while DM will not.

Second, we raise the possibility of information acquisition. A primary motivation for this paper is that asymmetric information concerning local preferences may exist between resource users and policy makers. This raises the possibility that the policy maker could collect information on spatial users' preferences in order to fine-tune the CP's management decisions. However, it should be noted that doing so could involve, for example, (costly) contingent valuation studies (Carson et al. 2001) that elicit conservation preferences (Alberini and Kahn 2009). Future research that utilized the framework developed above to measure the benefits of information gathering weighted against the costs could prove beneficial.

Third, we have assumed that the central planner has perfect information regarding resource characteristics. A promising extension would be to allow local users to have better resource information than the central planner. We conjecture that this would raise the efficiency of DM.

Finally, we consider the possibility of coordination among spatial property right owners. While the key limitation of the Centralized Planner is incomplete information, the key limitation of Decentralized Management is the lack of coordination over spatial externalities. But provided transaction costs are sufficiently low, this could be overcome via Coasian bargaining or profit sharing (Wiggins and Libecap 1985; Libecap and Wiggins 1985; Kaffine and Costello 2011). Because it seems plausible that transaction costs are prohibitively high (particularly when N is large), we have focused on non-cooperation.

But even if a profit sharing mechanism were possible to implement, it can be shown that such a mechanism would fail to reproduce the first best in the presence of conservation preferences. This result arises because pooling profits ignores the conservation margin of utility.[20] Recovering FB would require that conservationists somehow contribute their (true) willingness to pay for conservation to the profit sharing pool. To our knowledge, this mechanism has never been analyzed; we regard this as a fruitful area of research.[21]

6 Conclusion

We have analyzed the relative merits of central planning versus decentralized management of natural resources and have compared their resource and welfare outcomes to those under first-best management. The first-best solution reveals an important tension in managing natural resources characterized by both spatial resource and preference heterogeneity. On one hand, spatial management rules need to reflect heterogeneous externalities arising from resource movement. But management rules must also account for differences in preferences, which may not be known by the regulator. We show that while *centralized planning* may adequately capture spatial externalities between resources, only the average user is truly satisfied with the management rule due to lack of the information by the central planner regarding local preferences (Hayek 1945; Oates 1999). By contrast,

[20] In fact, decentralized management under profit sharing can lead residual stocks to *exceed* the first-best, as local conservation benefits are not shared while fishery profits are, and thus profit sharing incentivizes inefficiently high conservation.

[21] One could also consider incentive schemes akin to the Falkinger Mechanism (Falkinger 1996; Falkinger et al. 2000), whereby spatial property right owners are rewarded or penalized based on their escapement decisions relative to their peers. Further research into designing such a mechanism in the context of spatial and dynamic natural resources may yield important insights for decentralized resource management.

decentralized management (such as spatial property rights) allows users to select private management rules reflecting precisely their preferences within each location, but will ignore any spatial externalities created across locations (Bhat and Huffaker 2007; Janmaat 2005). A concrete policy implication of our results is that if the primary challenge facing natural resource management is resource heterogeneity (and the resulting spatial externalities), then *centralized planning* may dominate *decentralized management*. However, if the primary challenge is differences in preferences of various users, then delegation under *decentralized management* may be the second-best management option.

As governments in both the developed and the developing world continue to seek ways to reduce the economic and environmental losses associated with the common pool, spatial management has become increasingly pursued. While the prior literature on fiscal and environmental federalism provide useful insights into questions of the optimal "scale" of policy in a natural resource context, intertemporal dynamics and spatial connectivity introduce new challenges. As such, the issues of natural resource federalism considered in this paper are likely to increase in importance and warrant further inquiry.

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A Proof of Proposition 1

Proof. Per Assumptions 1 and 2 and by using residual stock ($\mathbf{e_t}$) (rather than harvest) as the control variable, this complicated dynamic optimization problem has a special structure, called "state independent control," for which the first-order conditions are independent of stock, x_{it} (Costello and Polasky 2008).[22] This allows us to separate the problem temporally, and implies that residual stock is location-specific, but time-independent (consistent with Proposition 1 in Costello and Polasky (2008)). This result accords with, but extends, existing resource models with perfectly elastic demand for which a bang-bang solution is implemented to achieve an optimal residual stock (see Costello et al. (2015)). Because optimal residual stock in patch i is constant, additional units of stock are simply harvested, so the shadow value on stock is just the value of an additional unit of harvest: $\frac{\partial V_{t+1}(\mathbf{x_{t+1}})}{\partial x_{jt+1}} = \alpha_j p_j \; \forall j$. The final term, $\frac{\partial x_{jt+1}}{\partial e_{it}}$ equals $f_i'(e_{it})D_{ij}$ by rewriting Equation 1 in terms of x_{jt+1} and differentiating with respect to e_{it}. Thus, what would otherwise be an extremely complicated spatial temporal optimization problem has a first order condition that compactly reduces from Equation 4 to:

$$-\alpha_i p_i + (1-\alpha_i)k + \delta \sum_{j=1}^{N} \alpha_j p_j f_i'(e_{it}) D_{ij} = 0 \quad \forall i. \tag{12}$$

Rearranging yields the residual stock rule in Proposition 1. □

B Proof of Corollary 1

Proof. We wish to show that $\frac{de_i^{FB}}{dD_{ii}} > 0$, $\frac{de_i^{FB}}{dD_{ij}} > 0$, $\frac{de_i^{FB}}{d\alpha_i} < 0$, and $\frac{de_i^{FB}}{d\alpha_j} > 0$. Let $\phi(D_{ii}, D_{ij}, \alpha_i, \alpha_j) = \frac{1}{\delta}\left[\frac{\alpha_i p_i - k(1-\alpha_i)}{\sum_{j=1}^{N} \alpha_j p_j D_{ij}}\right]$, which is the right-hand side of Equation 5. Re-

[22]If harvest was the control, then to achieve a desired residual stock would require a state-dependent control by the identity $e_{it} \equiv x_{it} - h_{it}$.

arranging Equation 5 such that $f_i'(e_i^{FB}) - \phi(D_{ii}, D_{ij}, \alpha_i, \alpha_j) = 0$, then by the implicit function theorem, $\frac{de_i^{FB}}{dD_{ii}} = \frac{\partial\phi/\partial D_{ii}}{f_i''(e_i)}$, and similarly for D_{ij}, α_i, and α_j. Thus, we have:

$$\begin{aligned}
\frac{de_i^{FB}}{dD_{ii}} &= -\frac{\alpha_i p_i(\alpha_i p_i - k(1-\alpha_i))}{\delta(\sum_{j=1}^N \alpha_j p_j D_{ij})^2 f_i''(e_i)} > 0 \qquad (13)\\
\frac{de_i^{FB}}{dD_{ij}} &= -\frac{\alpha_j p_j(\alpha_i p_i - k(1-\alpha_i))}{\delta(\sum_{j=1}^N \alpha_j p_j D_{ij})^2 f_i''(e_i)} > 0\\
\frac{de_i^{FB}}{d\alpha_i} &= \frac{(p_i + k)(\sum_{j\neq i}^N a_j p_j D_{ij}) + k p_i D_{ii}}{\delta(\sum_{j=1}^N \alpha_j p_j D_{ij})^2 f_i''(e_i)} < 0\\
\frac{de_i^{FB}}{d\alpha_j} &= -\frac{p_j D_{ij}(\alpha_i p_i - k(1-\alpha_i))}{\delta(\sum_{j=1}^N \alpha_j p_j D_{ij})^2 f_i''(e_i)} > 0
\end{aligned}$$

□

C Proof of Lemma 1

Proof. Rewriting utility to show the dependence on α_i, $U_{it}(x_{it}, e_{it}; \alpha_i)$, reveals that it is linear in the random variable, α_i, so $E_{\alpha_i}\left[U_{it}(x_{it}, e_{it}; \alpha_i)\right] = U_{it}(x_{it}, e_{it}; \bar{\alpha})$. □

D Proof of Proposition 2

Proof. By Lemma 1, the expected utility when only the distribution of α_i is known is equivalent to the utility of the expected value $\bar{\alpha} \equiv \sum \alpha_i/N$. Thus, the Dynamic Programming Equation is given by:

$$V_t(\mathbf{x_t}) = \max_{\mathbf{e_t}} \sum_{i=1}^N (\bar{\alpha} p_i(x_{it} - e_{it}) + (1-\bar{\alpha})k e_{it}) + \delta V_{t+1}(\mathbf{x_{t+1}}) \qquad (14)$$

with first-order conditions:

$$-\bar{\alpha} p_i + (1-\bar{\alpha})k + \delta \sum_{j=1}^N \frac{\partial V_{t+1}(\mathbf{x_{t+1}})}{\partial x_{jt+1}} \frac{\partial x_{jt+1}}{\partial e_{it}} = 0 \quad \forall i \qquad (15)$$

By the proof for Proposition 1, the first-order conditions can be expressed as:

$$-\bar{\alpha} p_i + (1-\bar{\alpha})k + \delta\bar{\alpha} \sum_{j=1}^N p_j f_i'(e_{it}) D_{ij} = 0 \quad \forall i. \qquad (16)$$

Rearranging yields the residual stock rule in Proposition 2. $\square$

E Proof of Corollary 2

Proof. The proof to part (a) hinges on the comparison of Equations 5 and 6. If $\alpha_i = \bar{\alpha}\ \forall i$, then the right-hand side of Equation 6 equals the right-hand side of Equation 5, which confirms the result.

To prove the inequality cases in part (b), note that $de_i^{FB} = \frac{de_i^{FB}}{d\alpha_i} d\alpha_i + \sum_{j \neq i} \frac{de_i^{FB}}{d\alpha_j} d\alpha_j$. From Corollary 1, $\frac{de_i^{FB}}{d\alpha_i} < 0$, and under Condition 1, $\frac{de_i^{FB}}{d\alpha_j} = -\frac{Q(\alpha_i p - k(1-\alpha_i))}{\delta(Q(N\bar{\alpha}-\alpha_i)+D\alpha_i)^2 f_i''(e_i)} > 0$. Because $\frac{de_i^{FB}}{d\alpha_j}$ is constant (under Condition 1) across all $j \neq i$, then $de_i^{FB} = \frac{de_i^{FB}}{d\alpha_i} d\alpha_i + \frac{de_i^{FB}}{d\alpha_j} \sum_{j \neq i} d\alpha_j$. Finally, if $\alpha_i > \bar{\alpha}$, CP is analogous to setting $d\alpha_i < 0$, which also implies that $\sum_{j \neq i} d\alpha_j > 0$, which allows us to unambiguously sign $de_i^{FB} > 0$. Thus, if $\alpha_i > \bar{\alpha}$, then $e_i^{CP} > e_i^{FB}$. The reverse is true if $\alpha_i < \bar{\alpha}$.

For part (c), set Equations 5 and 6 equal for all i and invoke Condition 1. Rearranging implies:

$$\frac{\alpha_i(p+k)-k}{Qp(N\bar{\alpha}-\alpha_i)+Dp\alpha_i} - \frac{\bar{\alpha}(p+k)-k}{Qp(N\bar{\alpha}-\bar{\alpha})+Dp\bar{\alpha}} = 0 \tag{17}$$

The left hand side is an implicit function that defines α_i as a function of all model parameters. The equivalent expression for a different patch, say j, is found by simply replacing α_i by α_j in Equation 17. Thus $\alpha_i = \alpha_j\ \forall i, j$, which implies that $\alpha_i = \bar{\alpha}\ \forall i$.

For part d), the Conditions ensure that all patches are symmetric in all aspects except α_i. From Equation 5, residual stock can be written as $e_i^{FB}(\phi(\alpha_i))$, suppressing the notation for other parameters. If $e_i^{FB}(\phi(\alpha_i))$ is convex, then by Jensen's Inequality, the residual stock at the average preference (under CP) is less than the average residual stock when considering the full range of preferences (FB). In order to show that $e_i^{FB}(\phi(\alpha_i))$ is convex, we first note that from the proof of Corollary 1, $\phi'(\alpha_i) > 0$ and $\phi''(\alpha_i) < 0$. Next, rearrange Equation 5 such that $f'(e_i^{FB}) - \phi(\alpha_i) = 0$. The total differential is given by $f''(e_i)de_i - \phi'(\alpha_i)d\alpha_i = 0$. Thus, $de_i = \frac{\phi'(\alpha_i)d\alpha_i}{f''(e_i)}$. Taking the total differential again gives $d^2e_i = -\frac{\phi'(\alpha_i)d\alpha_i f'''(e_i)}{(f''(e_i))^2}de_i + \frac{\phi''(\alpha_i)d\alpha_i}{f''(e_i)}d\alpha_i$. Substituting de_i from above gives:

$$\frac{d^2e_i}{d\alpha_i^2} = \frac{-(\phi'(\alpha_i))^2 f'''(e_i)}{(f''(e_i))^3} + \frac{\phi''(\alpha_i)}{f''(e_i)} > 0 \tag{18}$$

and thus $e_i^{FB}(\phi(\alpha_i))$ is convex, which proves the result. □

F Proof of Proposition 3

Proof. We assume that all model parameters, contemporaneous residual stocks, and contemporaneous stocks are common knowledge to all patch owners. Similar to Kaffine and Costello (2011), we consider a dynamic Cournot-Nash model in which owners simultaneously choose residual stocks in period t knowing that this procedure will be repeated every year into the future. Following the classic paper by Levhari and Mirman (1980), we solve for the subgame perfect Nash equilibrium by analytical backward induction on the Bellman equation for each owner i.

We proceed by backward induction for each patch owner. At the end of time the value function is zero: $V_{iT+1} = 0$ for all i. Thus the period T Bellman equation for owner i is simply

$$V_{iT}(\mathbf{x_t}) = \max_{e_{iT}} \alpha_i p_i (x_{iT} - e_{iT}) + (1 - \alpha_i) k e_{it} \tag{19}$$

whose interior solution is straightforward: $e_{iT}^* = 0$, as $\alpha_i p_i > (1-\alpha_i)k$. In the final period, each patch owner finds it optimal to harvest his entire stock, regardless of decisions made by other patch owners. Note that the patch-i value function has an analytical solution:

$$V_{iT}(\mathbf{x_t}) = \alpha_i p_i x_{iT} \tag{20}$$

which simplifies analysis in the penultimate period. Employing this result, the period $T-1$ patch i Bellman equation is:

$$\begin{aligned} V_{iT-1}(\mathbf{x_{T-1}}) &= \max_{e_{iT-1}} \alpha_i p_i (x_{iT-1} - e_{iT-1}) + (1-\alpha_i) k e_{iT-1} + \delta \alpha_i p_i x_{iT} \\ &= \max_{e_{iT-1}} \alpha_i p_i (x_{iT-1} - e_{iT-1}) + (1-\alpha_i) k e_{iT-1} + \delta \alpha_i p_i \sum_j f_j(e_{jT-1}) D_{ji} \end{aligned} \tag{21}$$

Taking e_{jT-1} as given (for $j \neq i$), the first order condition for owner i implies

$$f_i'(e_{iT-1}) = \frac{1}{\delta} \left[\frac{\alpha_i p_i - k(1-\alpha_i)}{\alpha_i p_i D_{ii}} \right] \tag{22}$$

Notice that this best response function for owner i is independent of both other owners' choices (e_{jT-1}) and of the state variable ($\mathbf{x_{T-1}}$). In other words, period $T-1$ decisions can be written as a set of pre-determined numbers, e^*_{1T-1}, e^*_{2T-1}, ..., that are independent of decisions made prior to period $T-1$.

This pattern turns out to hold in all preceding periods, and following Kaffine and Costello (2011) it is the case that the solution in all previous time periods is equal to Equation 22. Because the optimal choice of e_{it} is independent of both e_{jt} (for $j \neq i$) and of $\mathbf{x_t}$, this is both an open loop and a feedback control rule.

What happens if owner l deviates, so e_{lt} is given by some value $\tilde{e}_{lt}$ where $f_l'(\tilde{e}_{lt}) \neq \frac{1}{\delta}\left[\frac{\alpha_l p_l - k(1-\alpha_l)}{\alpha_l p_l D_{ll}}\right]$? There may be two effects on owner i's choices. First, it may affect his period t choices. Second, because future stock depends on owner l's period t choice, it may affect owner i choices in periods $t+1$, $t+2$, ... We showed above that e_{it} was independent of period t choices by all other patch owners, so we can rule out contemporaneous effects on patch owner i. But we also showed that in *any* period $t < T$, the optimal choice for owner i was independent of the state $\mathbf{x_t}$, which is the only conduit through which $\tilde{e}_{lt}$ affects owner i into the future. Thus, the deviation by owner l has no effect on owner i's future choices. Thus, under the assumptions of this model: (1) patch owner i's best response in period t is independent of period t choices by other patch owners and (2) patch owner i's optimal choice of residual stock in period $t+1$ is independent of choices made by *any* owner prior to period t.

With the time and patch independence established, we now return to the Dynamic Programming Equation for patch i under decentralized management:

$$V_{it}(\mathbf{x_t}) = \max_{e_{it}} \alpha_i p_i (x_{it} - e_{it}) + (1-\alpha_i) k e_{it} + \delta V_{it+1}(\mathbf{x_{t+1}}) \tag{23}$$

with first-order condition for patch i of:

$$-\alpha_i p_i + (1-\alpha_i)k + \delta \sum_{j=1}^{N} \frac{\partial V_{it+1}(\mathbf{x_{t+1}})}{\partial x_{jt+1}} \frac{\partial x_{jt+1}}{\partial e_{it}} = 0. \tag{24}$$

By the above, the best response function for any given patch owner is independent of other patch decisions. As such, the first-order conditions can be expressed as:

$$-\alpha_i p_i + (1-\alpha_i)k + \delta \alpha_i p_i f_i'(e_{it}) D_{ii} = 0 \quad \forall i. \tag{25}$$

Rearranging yields the residual stock rule in Proposition 3. □

G Proof of Corollary 3

Proof. The proof for part (a) follows from the comparison of Equations 5 and 7. Setting $D_{ij} = 0, \forall j \neq i$ in Equation 5, the right-hand side of Equation 5 is identical to the right-hand side of Equation 7, and residual stock in patch i under DM is equivalent to FB. To prove necessity, suppose $D_{ij} > 0$ for some $j \neq i$. Then the two expressions differ (the numerators are equivalent, but denominators differ).

The proof for part (b) also follows from the comparison of Equations 5 and 7. By Corollary 1, $\frac{de_i}{dD_{ij}} > 0$, and thus DM (equivalent to selecting e_i as if $D_{ij} = 0$) leads to strictly less residual stock than FB.

The proof for part (c) follows from the fact that as long as any patch k has $D_{kl} > 0$ for $k \neq l$, residual stock under DM in that patch will be strictly less than FB, and thus global residual stock under DM will be strictly less than FB. $\square$

H Proof of Proposition 4

Proof. The proof for part (a) follows from Corollaries 2c and 3a. The proof for part (b) follows from Corollaries 2a and 3b. $\square$

I Proof of Proposition 5

Proof. Define $\varepsilon_i = \alpha_i - \bar{\alpha}$ as the measure of preference heterogeneity for patch i. For part (a), because aggregate welfare over space and time is a continuous function of continuous functions of D_{ij}, aggregate welfare is continuous in D_{ij}. By Proposition 4, for any level of preference heterogeneity ($\varepsilon_i > 0$ for some i), welfare under DM is equivalent to FB when $D_{ij} = 0, \forall i \neq j$, and both strictly dominate CP. Because $DM \succ CP$ when $D_{ij} = 0, \forall i \neq j$, then by continuity, within a local neighborhood there exists a strictly positive level of out-dispersal $D_{ij} > 0$ for some $i \neq j$ where $DM \succ CP$.

For part (b), because aggregate welfare over space and time is a continuous function of continuous functions of α, total welfare is continuous in α and thus ε. By Proposition 4, for any level of out-dispersal $D_{ij} > 0$ for some $i \neq j$, welfare under CP is equivalent to FB when $\varepsilon_i = 0, \forall i$, and both strictly dominate DM. Because $CP \succ DM$ when $\varepsilon_i = 0, \forall i$, then by continuity, within a local neighborhood there exists a strictly positive level of preference heterogeneity ($\varepsilon_i > 0$ for some i) where $CP \succ DM$. $\square$

J Proof of Proposition 6

Proof. With the change to the utility function in Equation 8, the Dynamic Programming Equations under FB, CP, DM are now respectively:

$$V_t(\mathbf{x_t}) = \max_{\mathbf{e_t}} \sum_{i=1}^{N} \left(\alpha_i p_i(x_{it} - e_{it}) + (1 - \alpha_i) \sum_{j=1}^{N} k e_{jt} \right) + \delta V_{t+1}(\mathbf{x_{t+1}}) \tag{26}$$

$$V_t(\mathbf{x_t}) = \max_{\mathbf{e_t}} \sum_{i=1}^{N} \left(\bar{\alpha} p_i(x_{it} - e_{it}) + (1 - \bar{\alpha}) \sum_{j=1}^{N} k e_{jt} \right) + \delta V_{t+1}(\mathbf{x_{t+1}})$$

$$V_{it}(\mathbf{x_t}) = \max_{e_{it}} \left(\alpha_i p_i(x_{it} - e_{it}) + (1 - \alpha_i) \sum_{j=1}^{N} k e_{jt} \right) + \delta V_{it+1}(\mathbf{x_{t+1}}).$$

Following the procedure in the proofs to Propositions 1, 2, and 3 yields the optimal residual stock rules in Proposition 6. □

K Proof of Corollary 4

Proof. For part (a), compare the residual stock rules for DM and FB in Proposition 6 with the residual stock rules in Propositions 1 and 3. Note that DM does not change her residual stock when adopting global preferences, so $e_i^{DM} = \hat{e}_i^{DM}$. Examining the numerator of the optimal residual stock for FB reveals that optimal residual stock always increases under global preferences, so $e_i^{FB} < \hat{e}_i^{FB}$. By Corollary 3b, $e_i^{DM} < e_i^{FB}$. Putting these statements together yields: $\hat{e}_i^{DM} = e_i^{DM} < e_i^{FB} < \hat{e}_i^{FB}$, which establishes the result.

For part (b), we proceed with a proof by example. We invoke Condition 1, and assume $\delta = 1$, and $f_i(e_i)$ is quadratic, so $f_i'(e_i) = A_i - B_i e_i$. We then compare the residual stock rules for CP and FB in Proposition 6 with those in Propositions 1 and 2. Evaluating the residual stock for patch i, these rules become:

$$\begin{aligned} f_i'(e_i^{FB}) &= \frac{\alpha_i p - k(1-\alpha_i)}{Qp(N\bar{\alpha}-\alpha_i)+Dp\alpha_i} \equiv \phi_i^{FB} && (27)\\ f_i'(\hat{e}_i^{FB}) &= \frac{\alpha_i p - Nk(1-\bar{\alpha})}{Qp(N\bar{\alpha}-\alpha_i)+Dp\alpha_i} \equiv \hat{\phi}_i^{FB} && (28)\\ f_i'(e_i^{CP}) &= \frac{\bar{\alpha} p - k(1-\bar{\alpha})}{Qp(N\bar{\alpha}-\bar{\alpha})+Dp\bar{\alpha}} \equiv \phi_i^{CP} && (29)\\ f_i'(\hat{e}_i^{CP}) &= \frac{\bar{\alpha} p - Nk(1-\bar{\alpha})}{Qp(N\bar{\alpha}-\bar{\alpha})+Dp\bar{\alpha}} \equiv \hat{\phi}_i^{CP} && (30) \end{aligned}$$

Thus, $e_i = \frac{A_i-\phi_i}{B_i}$ for the ϕ's defined above. Consider a patch i for which $\alpha_i < \bar{\alpha}$, which implies that $e_i^{CP} < e_i^{FB}$ and $\hat{e}_i^{CP} < \hat{e}_i^{FB}$.[23] Define Δ_i as the difference in the difference in escapements under FB and CP under global versus local preferences, such that $\Delta_i = (\hat{e}_i^{FB} - \hat{e}_i^{CP}) - (e_i^{FB} - e_i^{CP})$. Both terms in parentheses are positive, so $\Delta_i > 0$ implies the wedge between escapements is larger under global preferences, while $\Delta_i < 0$ implies the wedge is smaller under global preferences. After some algebraic simplification,

$$\Delta_i = \frac{pk(\alpha_i - \bar{\alpha})((N-1)(Q-D)+DN\bar{\alpha})}{B(Qp(N\bar{\alpha}-\alpha_i)+Dp\alpha_i)(Qp(N\bar{\alpha}-\bar{\alpha})+Dp\bar{\alpha})} \tag{31}$$

The denominator is unambiguously positive, so the sign hinges on the numerator. The first term in parenthesis $(\alpha_i - \bar{\alpha})$ is negative given the assumption $\alpha_i < \bar{\alpha}$. This negative term is multiplied by the second term in parenthesis $((N-1)(Q-D)+DN\bar{\alpha})$, which can be positive or negative. For example, if $Q = D$, then this term is positive and $\Delta_i < 0$, while if $Q = 0$ and $\bar{\alpha} < \frac{N-1}{N}$, then this term is negative and $\Delta_i > 0$. Thus, $\Delta_i \lesseqgtr 0$, and central planning residual stock in patch i may be closer or farther from first-best.

□

[23]That $e_i^{CP} < e_i^{FB}$ follows from Corollary 2b. That $\hat{e}_i^{CP} < \hat{e}_i^{FB}$ can be confirmed by subtracting Equation 30 from 28 and noting that non-negativity of the numerator of Equation 30 requires $p \geq Nk(1-\bar{\alpha})/\bar{\alpha}$.