Strategic Rule Formation Under Externalities

Multilevel Institutional Analysis Using Network Formation Games

Luke M. Shimek¹

Abstract: This paper provides a basic example of the use of network formation games (NFGs) as a formal modeling tool inside the institutional analysis and development (IAD) framework. Network formation games are a relatively recent research area within the broader study of networks and are able to maintain the mathematical integrity of standard strategic game-theoretic models while making the structure less linear and removing as many restrictions as possible.

The impetus behind using network formation games in the IAD framework is their ability to readily scale up into supernetworks, in which the nodes are a particular network formation, and the arcs may be Markov chains, dictating transitions between networks. This supernetwork can then act as a node in a supersupernetwork, etc. This is highly useful, since the IAD framework is interested in multi-level analysis.

This article addresses a simple public bad game: there are three players, each with 1 bag of garbage, and the government disposal/recycling official. The players decide whether to pass their garbage to one of their neighbors at no cost, or to send it to the disposal/recycling center for a diminishing cost per bag. In the base game, there is no side payments between individuals and the players act unilaterally, thus the results are either socially suboptimal or are borne by one or two of the agents. At the collective choice level, players can vote whether they would prefer side payments or to maintain the status quo. Finally, the analysis considers the constitutional level, where the voting procedures are identified (unilateral, majority, or supermajority).

I find the greatest improvement to social welfare results from a coalition assigning a Pigouvian tax to create side-payments between players. Changing the voting from unilateral to majoritarian to super majoritarian generally better distributes the cost of littering, but only weakly lowers the level of trash. However, more important than the results, this model demonstrates a promising future connection between network formation games and the IAD framework.

¹ PhD. student, Joint Public Policy: School of Public and Environmental Affairs and Department of Political Science; The Vincent and Elinor Ostrom Workshop in Political Theory and Policy Analysis, Indiana University- Bloomington. lshimek@indiana.edu Draft Date: 2/13/2013

I. Introduction

Externalities are the bane of markets. An externality exists when the costs or benefits of an action are not completely born by the initiating actor and pass on to others in society who did not choose the original action. In general, their presence implies that the Fundamental Welfare theorems do not hold and that prices do not signal enough information to either consumers or producers. Efficiency or Pareto optimality are not present in the market, causing market failure. When an institution such as a market fails, actors turn to other institutions, such as the state, the commons, or private clubs to provide the appropriate level of a good. Thus, the study of externalities is entwined with the study of institutions.

The study of institutional interaction dates back to Plato's *The Republic*. Throughout history, institutions have been studied as tools for wielding power over other individuals, to symbolic and semi-religious structures that culturally bound society. The approach to institutions in this paper reflects the work done in New Institutional Economics (Coase 1937, Williamson 1975) and in the Institutional Analysis and Development framework literature (Ostrom 2007). Specifically, institutions are treated as the rules of the game (North 1991), which affect actors from multiple levels.

This paper studies how changing network formation rules affect the outcomes for the agents and the environment as a whole. Specifically, in a voting situation, can giving government agents a vote of their own alter outcomes, and does changing the number of votes required to take an action reduce the level of the externality? Given the assumptions of the model, the environment is only mildly better off giving the government agent a vote. The major benefit the government can offer is a Pigouvian tax that creates side-payments to lower trash levels. Changing the voting from unilateral to majoritarian to super majoritarian generally better distributes the cost of littering, but only weakly lowers the level of trash in the environment. More important than these standard results, the model presented in this paper demonstrates the promising future connection between network formation games and the IAD framework.

The paper proceeds as follows: Section II presents a brief overview of the IAD framework. Section III reviews some of the important literature in network formation games, what networks contribute to the broader game theoretic literature, and why network formation games represent a simple way of applying the IAD framework to theoretical questions. Section IV delves into the example: a trash and recycling game. Section V provides some conclusions and extensions for further research.

II. The Framework: Institutional Analysis and Development

The Institutional Analysis and Development (IAD) framework has been in development over the years at the Ostrom Workshop in Political Theory and Policy Analysis at Indiana University.

This section further discusses these four characteristics of the IAD framework inside an 'action situation'. An action situation is an analytical construct that allows researchers to focus on the immediate structures, which affect decision making and thus outcomes in a particular scenario.

Action situations are similar to a game in game theory.² What follows is simply a 'quick and dirty' paraphrase of thirty years' worth of work.

Individuals are the basic unit of analysis in the IAD framework. They are both the actors and those upon whom actions are taken. Individuals are generally assumed to have at least an ordinal preference ordering, some basic understanding of the consequence of their actions, and limited uncertainty about the actions of those around them. Those using the IAD framework take Herbert Simon's notion of bounded rationality seriously, where actors do not have complete and perfect information about the present situation and the payoffs to their actions (Simon 1955). However, the framework does assume that an individual acts rationally based off the information they do have at their disposal.

As mentioned above, individuals act based on their preferences over the incentives provided in a given situation. These incentives can take many forms, ranging from personal monetary to culturally appropriate to what is best for future generations of their family or clan. One can see that being rationally self-interested still applies to these individuals, as long as one dismisses the notion that being self-seeking can be quantified as often occurs in economic and political game theory. Inside an action situation, outside conditions, such as the physical environment, history, the community, and the institutional environment, determine the incentives facing the actors.

Institutions are the rules governing the actions of individuals in all situations. They are created by individuals seeking to solve collective action problems by either dictating (i.e. 'must' or 'must not' rules) or promoting ('may' rules) some set of actions.³ IAD researchers are interested in *de facto* rules as opposed to *de jure* rules. Another attribute of institutions is that they exist at multiple levels. The IAD framework identifies four levels of institutional collective-choice, constitutional, meta-constitutional. Each successive institutional level informs the prior. In the US, the meta-constitutional level is the state itself, the constitution, the collective-choice level would be the government as written in the constitution, the collective-choice level would be the government and laws it passes, and the operational level is the interaction between individuals and government agencies on a regular basis. Seeing as most of these levels are made up of individuals, actors at given institutional level can affect change at the deeper institutional level, though these changes are more costly and time consuming to change.

The IAD framework results from a unique form of inquiry. It stems from the belief that no method or practice, no matter how robust, is a panacea. Therefore, associates of the Workshop are interdisciplinary in nature with a multitude of theories and methods for understanding institutions (or the world in general). In this vein, it became important to attempt to create uniform terminology

² For a more in-depth study and visualizations of the frameworks setup, see **Ostrom, E.** 2005. Understanding Institutional Diversity. Princeton: Princeton University Press, _____. 2007. "Institutional Rational Choice: An Assessment of the Institutional Analysis and Development Framework," P. A. Sabatier, *Theories of the Policy Process.* Boulder, CO: Westview Press, 21-64, **McGinnis, M. D.** 2011. "An Introduction to Iad and the Language of the Ostrom Workshop: A Simple Guide to a Complex Framework." *Policy Studies Journal*, 39(1), 169-183.

³ This contrasts with culturalist, Marxist and several other views of institutions as myths and ceremonies, elites' tool of repression, and a myriad of other functions.

for the study of institutions. Hence, frameworks, theories, and models are three distinct tools. Frameworks are nested sets of components that can describe human behavior, but do not contain or imply causality. Theories are approaches using frameworks, which begin to impose causal relationships between components in a framework. Models are explicit in the relationships between components which results in testable hypotheses. This paper presents a formal analytical model from a network game theoretical approach from within the IAD framework. The next section discusses the theory, and then Section IV builds the model.

It is important to note that the model presented here is only a baseline for explaining behavior in the real world. The IAD approach to inquiry demands that this merely be a starting place for studying phenomena. Consequently, much of the Workshop's inquiry has been in to the real world study of the commons, which Garrett Hardin (1968) theorized as a tragedy. However, Workshop associates have taken his theory to the real world and show that it is incomplete; the assumptions underlying the tragedy of the commons do not hold but in a small case of real world situations, and this form of inquiry has led to the development of the socio-ecological systems approach to the commons which attempts to identify both broad and narrow categories that help determine the drama of the commons (Ostrom 2009).

III. The Theory: Network Formation Games

A network can be a form of organization, a means of relaying people (e.g. trains, planes, and automobiles) or information (e.g. the internet), or a policy tool used in New Public Management (Agranoff and McGuire 2001). Networks can be an analytical tool as well. Researchers have used a combination of networks and complex systems analysis to study the possible spread of a pandemic across the globe.⁴ Various social scientists and policy analysts have adopted the term as a modifier to, or replacement for the term social capital. In that case, networks provide a residual explanation for unexplained phenomena in a social theory.⁵ Considering two basic components, nodes and arcs, compose all networks, an intelligent person could find networks to be omnipresent.

While computer scientists, engineers, political scientists, public administration analysts, sociologists, and statisticians have been working formally with network theory since the early 1960s, economists have shown up late to the game. The first truly economic analysis of was conducted by Roger Myerson (1977) where he looked at the relationships between graphs and cooperative game theory. Prior to this point, most formal network theory focused on deterministic models, particularly those in complex systems analysis. In his paper, one can see the beginning of what economists could contribute to network theory, namely the strategic formation of networks by rational agents.

Network formation games are a relatively recent research area within the broader study of networks. Two groundbreaking works in the field were Jackson and Wolinsky (1996) and Bala and

⁴ Much of this research has occurred in the Center for Complex Networks and Systems Research at Indiana University. See http://cnets.indiana.edu for other topics the center is currently addressing.

⁵ An undercurrent in some of the less academic literature leaves the impression that network theory can be a mindset or worldview, and this view is often associated with support for democracy and democratic ideals.

Goyal (2000). These papers are interested in seeing how, given a particular set of rules, agents strategically form links between each other. Between the two papers, it is easy to note that different formation rules result in different types of networks forming. These two papers accelerate the trend of working with network formation games in economics. Recent work by Page and Wooders (2007b) has moved economists from strictly studying homogeneous linking networks, where the arcs are non-directional and all the same type, to studying heterogeneous directed networks, where the arcs have a specific direction and may be of multiple types and intensities.

The goal behind network formation games is to take the vast knowledge of strategic network games from standard game theoretical literature and (a) attempt to make the structure less linear (b) and remove as many assumptions as possible while (c) maintaining the mathematical integrity of the game theoretic models. As Section IV demonstrates, the degree of complexity in relationships is maintained from standard game theory while the degree of complexity in the mathematics is not. Furthermore, network formation games see equilibrium as only one possible outcome of a game and instead focus on the dynamics of stability for multiple equilibria (Page et al. 2005). Finally, all of these properties lead to rigorous modeling that can be quickly comprehended visually via graphs and tables.

Several factors contribute to the beneficial blending of the IAD framework and Network Formation Games. First, as mentioned in the previous paragraph, network formation games can maintain some of the complexity of an action situation while still allowing the researcher to clearly analyze the components and dynamics. Second, network formation games do not presuppose strict rationality; a thin version is acceptable. In fact, network formation games could model bounded rationality and learning as types of arcs could change between different levels in a network (e.g. A student knows what the persons next to her are going to do, but she only has a vague sense of what the remaining individuals in the room plan to do). Third and most importantly, network formation games have the ability to readily scale up into supernetworks, in which the nodes are a particular network formation, and the arcs may be Markov chains or in the case of the game in this paper, strategic network rules, dictating transitions between networks. This supernetwork can then act as one node in a super- supernetwork, etc. This is highly useful, since the IAD framework is interested in multi-level analysis. It is hypothetically possible to start with a network of an individual weighing the probabilities of certain states of behavior, scaling it up to an operational situation where that individual interacts with others in operational situations, and continuing to scale up to the metaconstitutional level.

The remainder of this paper attempts to demonstrate these strengths of network formation games operating under the auspices of the IAD framework through a basic example.⁶ The example is an extension of the trash game first posed in Shapley and Shubik (1969). They showed that given an externality, the bag of garbage one throws into a neighbor's yard, and more than two players, there is no core to the game and hence no equilibrium. The game has been recently re-analyzed in

⁶ A good example of network formation games modeling agent behavior with club goods is **Page, J., Frank H. and Wooders, M. H.** 2007a. "Club Networks with Multiple Memberships and Noncooperative Stability," *Conference in Honor of Ebud Kalai.*

networks literature and shown that there are both farsightedly and Nash stable sets in the network (Page et al. 2002). Here, the analysis is extended to include a government agent in charge of recycling. In addition, agents have voting options and strategic behavior at the operational, collective-choice, and constitutional levels.

IV. The Game: Trash & Recycling Amongst Three Players

The basic action situation proceeds as follows: on a small cul-de-sac, three neighbors must figure out what to do with their trash. Each neighbor has three options: (1) he can take his trash to the recycling official in town, who charges a recycling fee; (2) he can dump his trash in one of his neighbor's yards, which is costless to himself; or (3) he can figure out his own way of disposing of the garbage, which is costly to himself, but generally less costly than taking the garbage to the recycling official. The neighbors do not initially know what the other neighbors are going to do, but they do know that they face the same cost structure.

A. Primitives

Network formation games rely on four primitives to populate the model: the feasible set of networks made up of arcs and nodes, players' preferences, the rules of network formation, and a dominance relation over feasible networks. The primitives offered here are specific to this game, which is a heterogeneous directed network; for an abstract approach, see Page and Wooders (2007b).

1. Feasible Networks

As mentioned above, the set of all feasible networks is composed of all arcs A, with a typical element defined as j, and all nodes N, with a typical element defined as i.

Definition 1 (Heterogeneous Direct Network (Page, Wooders and Kamat 2005))

A heterogeneous directed network, G, is a subset of $A \times (N \times N)$. Given any $G \subseteq A \times (N \times N)$, each ordered pair $(j,(i,i')) \in G$ consisting of an arc type and an arc is called a labeled arc in G. The collection of all labeled directed networks is denoted by $P(A \times (N \times N))$.

The ordered pair j(i,i') can be read as an arc of type j proceeding from i to i'. In this paper, $N = \{i_1, i_2, i_3, i_R\}$ and $A = \{j_1, j_2, j_3, j'_1, j'_2, j'_3, j'_R\}$ which will be ascribed qualities later. Thus, $G_1 = \{j_1(i_1, i_1), j_2(i_2, i_1), j_3(i_3, i_1)\}$ is an example of a network in this action situation; Figure 1 depicts the heterogeneous directed network G_i . Note that loops are a legitimate type of arc and that a loop is not equivalent to not having any arcs at all.

FIGURE 1: HETEROGENEOUS DIRECTED NETWORK G_1



The feasible network of the action situation G is the set of all possible G. See Table 1 for the feasible set G of the game.

2. Preferences

Let d denote the set of players with typical element denoted by p. Note that in this action situation, the set of players d who can create arcs is not the same as the set of nodes N. Players can have either strong or weak preferences.

Definition 2: (Strong and Weak Preferences (Page and Wooders 2007b))

For each player $p \in d$ let \succ_d be an irreflexive binary relation on G and write $G' \succ_d G$ if player $p \in d$ strongly prefers network $G' \in G$ to network $G \in G$. For weak preferences, for each player $p \in d$ let d be a binary relation on G and write $G' \succ_p G$ if player $p \in d$ either strongly prefers network $G' \in G$ to network $G \in G$ or is indifferent between G' and G. This latter case is weak preferences.

Similarly, players can form coalitions that have weak preferences, where at least one player prefers G' to G and the others are indifferent.

In this example, the preferences are formed based on the real-valued network payoff function, $\{v_p(\cdot)\}_{p \in d}$. Given this payoff function, player *p* prefers network *G'* to *G* if $v_p(G') > v_p(G)$ and weakly prefers network *G'* to *G* if $v_p(G') \ge v_p(G)$. Again, players can form coalitions that would have both strong and weak preferences based on $v_p(\cdot)$.⁷

3. Network Rules

A major element of this example is the changing of network formation rules to optimize outcomes, thus only the base rule is defined here as a slight modification of Bala and Goyal (2000)'s non-cooperative or unilateral-unilateral rules: (a) arc addition is unilateral and is carried out by only the initiator, player p; also, (b) arc subtraction from player p to p' is an unilateral action of player p. Thus, player p can change network G to network G' without regard to player p''s preferences. Later, voting rules will be implement that will require the formation of coalitions. The ability to move from

⁷ Abstractly, one can view each network G in feasible set G as a node in a larger network, thus representing coalitional preferences as a heterogeneous directed network.

network to network is called an effectiveness relation, \rightarrow_s where *S* is a coalition of players. Thus, if $G \rightarrow_s G'$, under the rules of network formation, the coalition can change network *G* to *G'* by adding or subtracting arcs. In this example, all players' or coalitions' private decision-making occurs prior to receiving the payoffs, which occurs once the network is formed.

4. Stability

Definition 3: Stability

off - and possibly worse off.

(a) (Nash Stable Networks (Page and Wooders 2007b))

A network is Nash stable if, whenever an individual player has the power to change the network to another network, the player will have no incentive to do so.

(b) ((Farsightedly Stable Networks (Page, Wooders and Kamat 2005))
 A network is farsightedly stable if no agent or coalition of agents is willing to alter the network (via the addition, subtraction, or replacement of arcs) for fear that such an alteration might induce further network alterations by other agents or coalitions that in the end leave the initially deviating agent or coalition no better

Here, stability introduces an alternative to equilibrium. In network formation games, equilibrium is a special case of stable sets, specifically when the stable set is of size 1. Instead, stable sets can also consist of circuits, where no network *G* dominates all other $G' \in \mathbf{G}$.⁸

While the recycling official i_R is not considered a player in the sense that she cannot form arcs, she still has preferences and affects the real-valued payoffs of the players. i_R preferences can be ranking, from highest to lowest, for players to give her their recycling, to dispose of their waste themselves, and lastly to give their trash to a neighbor. The recycling official also experiences economies of scale in recycling. Thus, the first bag of garbage received is expensive to process, but all subsequent bags are cheap.

The real-valued payoffs are as follows: each bag of garbage a player ends up with costs 2 utils. It costs 0 to give the bag to a neighbor; to give the garbage to the recycling official costs 3 utils for the first bag and each subsequent bag costs 1.

B. Simultaneous Gameplay

In this scenario, players decide and take action at the same time; in addition, because of the simultaneity of play, the recycling official is able to evenly distribute the cost of recycling among all players who chose to recycle.

1. Operational Level

Table 1 has all feasible networks of the game and Table 2 has the feasible payoffs. It is important to note that it is more costly, but individually and socially, for only one player to go to the recycling

⁸ For more on the existence and non-emptiness of a stable core/network, see **Dutta, B. and Mutuswami, S.** 1997. "Stable Networks." *Journal of Economic Theory*, 76(2), 322-344. and **Chwe, M.** 2000. "Communication and Coordination in Social Networks." *Review of Economic Studies*, 67(1), 1-16.

official. Looking at the unilateral rules for farsighted stability in Table 3 (illustrated in Figure 2), shows that in over half the cases, each player ends up with one bag of garbage. Of the remaining cases, only one results in the socially optimal solution, and this case can only be arrived at stochastically (i.e. players would have to start in that situation). Moreover, changing to Nash stability (Table 3 and Figure 3) eliminates the socially optimal case. Notice that payoffs were symmetric in the farsightedly stable case, but this does not necessarily hold in the Nash case.

2. Collective-Choice Level

At the collective-choice level of analysis, the players can decide whether the recycling official's opinion should count towards the actions of the network. Obviously, having more people who can vote does not affect the strategies and outcomes in a non-cooperative game, thus players are indifferent between allowing i_R to vote or not vote, as demonstrated in Figure 4.

Preempting the analysis on constitutional level change, allowing the recycling official to vote in a majoritarian or super majoritarian voting system can affect both farsightedly and Nash stable networks. In the majoritarian case, having four voters instead of three requires a coalition of size three to have an effectiveness relation on the supernetwork. However, this coalition size is already required for super majoritarian voting, thus players are indifferent over i_R voting or not voting. Figure 4 illustrates these results. Since there is at least one case where having i_R vote improves outcomes (Case 1 versus Case 2 in Tables 4 and 5) and in no cases does it reduce payoffs, if the type of voting is unknown, players will chose to have i_R vote. This holds for both Nash and farsighted stability.

3. Constitutional Level

At the constitutional level, players determine how decisions will be made at the lower institutional levels. Here, players choose between three voting types: unilateral, majoritarian, and super majoritarian. Unilateral voting is the baseline rules presented in the network rules section. Majoritarian voting requires a coalition of greater than fifty percent for an arc to be added or subtracted. Super majoritarian voting requires a coalition of greater than two-thirds for an arc to be added or subtracted. Moreover, the voting rules require the sending player to propose the potential arc and to be able to form a coalition that weakly prefers said arc. If the proposed arc fails to find a winning coalition, the sending player continues to propose prospective arcs until a winning coalition is found.

Tables 4 and 5 (illustrated in Figures 2 and 3) show the farsightedly and Nash stable sets, resp. As noted above, the stable networks in majoritarian voting alternate between Case 1 and Case 2 (in Tables 4 and 5) based on collective-choice level decisions. Notice, however, that in super majoritarian scenarios, the social optimality is achieved for Nash stability, and is one of two possibilities in the case of farsighted stability (thus having a lower expected cost than Case 1). Thus, if the type of collective-choice level institution is unknown, players prefer super majoritarian to majoritarian to unilateral voting. See Figure 7 for an illustration of constitutional supernetwork movement.

C. Sequential Gameplay

As a slightly different scenario, the game was changed to have sequential gameplay. Play starts with i_1 choosing and forming an arc; i_2 observes this action and chooses/forms his arc accordingly; i_3 observes both i_1 and i_2 's arcs and plays last. Payoffs are then allocated. Because actions take place sequentially, the recycling official cannot immediately make side-payments.

1. Operational Level

Table 1 has all feasible networks of the game and Table 6 has the feasible payoffs (note that payoffs are different from Part B). A similar process as backward induction in standard game theory determines the stable sets. Since i_3 acts last, the other players take his payoffs into account; seeing that his optimal arc choices are $j_3(i_3,i_1)$ or $j_3(i_3,i_2)$ with cost of zero to i_3 . For the sake of simplicity, assume that indifference between actions leads to alternating between choices evenly. Next, i_2 choices $j_2(i_2,i_1)$ as it offers the lowest expected cost over all choices by i_1 . Given these prior choices, i_1 minimizes his expected cost by choosing $j_1(i_1,i_3)$, which results in the set of networks $[G_3, G_{19}]$. Notice that the network set $[G_9, G_{25}]$ off the same expected costs, thus being farsightedly stable, but i_1 has a better immediate deviation strategy, and thus this is not Nash stable. Figure 5 presents the stable sets.

2. Collective-Choice Level

This is where the action situation gets particularly interesting. Instead of the recycling official being able to vote, players can decide whether they want her to allocate side payments. Tables 7 and 8 contain the stable networks and Table 9 contains the payoffs. With the introduction of an additional case, there also exists an additional case in Table 9. Here, the introduction of voting by i_R removes the last mover advantage from i_3 in the cases of majoritarian and super majoritarian voting, so while the socially optimal is not reached, there is an equitable distribution of costs. The key to reaching the social optimum in this case is the official's ability to distribute side payments, represented by cases 3 and 4. In case 3, because i_3 can operate in the same manner as the baseline case, i_1 and i_2 have to make sure he has not costs; whereas in case 4, because of the ability of i_R to ensure equitable incomes, that threat point is removed and the players split the social savings of sending the trash to the recycling center. Figure 6 outlines the movements between collective-choice rules. In the unilateral case, there is a circuit between side-payments with no voting and side-payments with voting, since i_3 can always move away from having to pay any costs. In the majoritarian cases, side-payments with voting becomes the stable point.

3. Constitutional Level

Supernetwork characteristics remain the same as in the simultaneous case, thus Figure 7 applies to this case as well. However, it is important to note that i_3 is worse off in both the majoritarian and the super majoritarian case. Either player i_1 or i_2 moving from U to the M or SM nodes drives the

dynamics. Once at either of these nodes, i_3 no longer has an effective relationship over the supernetwork.

V. Implications & Extensions

The implications of the model are straightforward and well-established in other theoretical literature. The more players are involved in the decision-making process, the better the networks perform socially. This merely reflects the internalization of the externality to the group as a whole. In addition, giving the government a voice does not necessarily improve social outcomes, but it does force a more equitable voting process and thus more equitable payoffs. Finally, the ability to make side-payments can result in a socially optimal outcome, without changing any of the other structures of the game. Side-payments in this game are essentially the equivalent of a Pigouvian tax, where the government subsidizes good behavior (turning in the trash to the recycling agent), and taxes poor behavior (passing the trash to a neighbor).

The main result of this paper is its ability to illustrate how network formation games dovetail nicely with the IAD framework, particularly the network attribute of supernetworks with the IAD attribute of nested levels of analysis. This fact is illustrated in Figure 8, which essentially places Figure 7 under a microscope, replacing the *M* node with the majoritarian case from the sequential example, then replacing 'allowing the recycling official to both vote and make side-payments' with the stable set of networks. In the terminology of the IAD framework, constitutional level rules affect a larger array of collective-choice institutions and rules, which affect the day-to-day handling of externalities.

The number of extensions are tremendous. Simple extensions would be adding additional players, changing the voting rules to represent that of the US Congress, or changing the technology for recycling. An extension that is more dramatic would be to include players indirectly linked to the initial set of players and whose knowledge of the initial players' actions or payoffs is incomplete. The goal of this article was to create an initial blueprint upon which richer research can be build.

Table 1: All Possible Garbage Game Networks

		$j_{3}(i_{3},i_{1})$		
	$j_2(i_2,i_1)$	$j_2(i_2,i_2,)$	$j_2(i_2,i_3)$	$j_2(i_2,i_R)$
$j_1(i_1,i_1)$	G ₁	G ₅	G ₉	G ₁₃
$j_1(i_1,i_2)$	G ₂	G_6	G ₁₀	G_{14}
$j_1(i_1,i_3)$	G ₃	G_7	G ₁₁	G ₁₅
$j_1(i_1,i_R)$	G_4	G_8	G ₁₂	G_{16}

•		/·	•	1
- 1	21	12	.1 2	
5	5		,• ₂	/

	$j_2(i_2,i_1)$	$j_2(i_2,i_2,)$	$j_2(i_2,i_3)$	$j_2(i_2,i_R)$
$j_1(i_1,i_1)$	G ₁₇	G ₂₁	G ₂₅	G ₂₉
$j_1(i_1,i_2)$	G ₁₈	G ₂₂	G ₂₆	G ₃₀
$j_1(i_1,i_3)$	G ₁₉	G ₂₃	G ₂₇	G ₃₁
$j_1(i_1,i_R)$	G ₂₀	G ₂₄	G ₂₈	G ₃₂

 $j_{3}(i_{3},i_{3})$

		92(2)2)		
	$j_2(i_2,i_1)$	$j_2(i_2,i_2,)$	$j_2(i_2,i_3)$	$j_2(i_2,i_R)$
$j_1(i_1,i_1)$	G ₃₃	G ₃₇	G ₄₁	G ₄₅
$j_1(i_1,i_2)$	G ₃₄	G ₃₈	G ₄₂	G ₄₆
$j_1(i_1,i_3)$	G ₃₅	G ₃₉	G ₄₃	G ₄₇
$j_1(i_1,i_R)$	G ₃₆	G_{40}	G ₄₄	G_{48}

 $j_{3}(i_{3},i_{R})$

		0		
	$j_2(i_2,i_1)$	$j_2(i_2,i_2,)$	$j_2(i_2,i_3)$	$j_2(i_2,i_R)$
$j_1(i_1,i_1)$	G ₄₉	G ₅₃	G ₅₇	G ₆₁
$j_1(i_1,i_2)$	G ₅₀	G ₅₄	G ₅₈	G ₆₂
$j_1(i_1,i_3)$	G ₅₁	G55	G ₅₉	G ₆₃
$j_1(i_1,i_R)$	G ₅₂	G ₅₆	G ₆₀	G ₆₄

		i 2 (i 2. i 1)		
	$j_2(i_2,i_1)$	$j_{2}(i_{2},i_{2},j_{3},j_{3})$	$j_2(i_2,i_3)$	$\mathbf{j}_2(\mathbf{i}_2,\mathbf{i}_R)$
$i_1(i_1,i_1)$	(-6,0,0)	(-4,-2,0)	(-4,0,-2)	(-4,-3,0)
$j_1(i_1, i_2)$	(-4,-2,0)	(-2,-4,0)	(-2,-2,-2)	(-2,-5,0)
$j_1(i_1,i_3)$	(-4,0,-2)	(-2,-2,-2)	(-2,0,-4)	(-2,-3,-2)
$j_1(i_1,i_R)$	(-7,0,0)	(-5,-2,0)	(-5,0,-2)	(-4,-2,0)
		į 3(i 3, i 2)		
	$j_2(i_2,i_1)$	$j_2(i_2,i_2,)$	$j_2(i_2,i_3)$	$j_2(i_2,i_R)$
$j_1(i_1,i_1)$	(-4,-2,0)	(-2,-4,0)	(-2,-2,-2)	(-2,-5,0)
$j_1(i_1,i_2)$	(-2,-4,0)	(0,-6,0)	(0,-4,-2)	(0,-7,0)
$j_1(i_1,i_3)$	(-2,-2,-2)	(0,-4,-2)	(0,-2,-4)	(0,-5,-2)
$j_1(i_1,i_R)$	(-5,-2,0)	(-3,-4,0)	(-3,-2,-2)	(-2,-4,0)
		j 3(i 3,i 3)		
	$j_2(i_2,i_1)$	$j_2(i_2,i_2,)$	$j_2(i_2,i_3)$	$j_2(i_2,i_R)$
$j_1(i_1,i_1)$	(-4,0,-2)	(-2,-2,-2)	(-2,0,-4)	(-2,-3,-2)
$j_1(i_1,i_2)$	(-2,-2,-2)	(0,-4,-2)	(0,-2,-4)	(0,-5,-2)
$j_1(i_1,i_3)$	(-2,0,-4)	(0,-2,-4)	(0,0,-6)	(0,-3,-4)
$j_1(i_1,\!i_R)$	(-5,0,-2)	(-3,-2,-2)	(-3,0,-4)	(-2,-2,-2)
		$j_{3}(i_{3},i_{R})$		
	$j_2(i_2,i_1)$	$j_2(i_2,i_2,)$	$j_2(i_2,i_3)$	$j_2(i_2,i_R)$
$j_1(i_1,i_1)$	(-4,0,-3)	(-2,-2,-3)	(-2,0,-5)	(-2,-2,-2)
$j_1(i_1,i_2)$	(-2,-2,-3)	(0,-4,-3)	(0,0,-7)	(0,-4,-2)
$j_1(i_1,i_3)$	(-2,0,-5)	(0,-2,-5)	(0,0,-7)	(0,-2,-4)

	Farsightedly Stabl	
	Recycler does not Vote	Recycler Votes
Unilateral	$\{G_7,G_{10},G_{19},G_{25},G_{34},G_{37},G_{48},G_{56},G_{61},G_{64}\}$	$\{G_7, G_{10}, G_{19}, G_{25}, G_{34}, G_{37}, G_{48}, G_{56}, G_{61}, G_{64}\}$
Majority	$\{G_7,G_{10},G_{19},G_{25},G_{34},G_{37},G_{48},G_{56},G_{61},G_{64}\}$	$\{G_{37},G_{64}\}$
Super Majority		ì
	{V37,V64} Nash Stable	$\{G_{37},G_{64}\}$
	(V37,V64) Nash Stable Recycler does not Vote	{G ₃₇ ,G ₆₄ } Recycler Votes
	1037,0641 Nash Stable Recycler does not Vote	{G ₃₇ ,G ₆₄ } Recycler Votes
Unilateral	$\{0_{37}, 0_{64}\}$ Nash Stable Recycler does not Vote $\{G_2, G_3, G_{10}, G_{11}, G_{18}, G_{19}, G_{26}, G_{27}\}$	$\{G_{37},G_{64}\}$ Recycler Votes $\{G_2,G_3,G_{10},G_{11},G_{18},G_{19},G_{26},G_{27}\}$
Unilateral Majority	$\{037,064\}$ Nash Stable Recycler does not Vote $\{G_2,G_3,G_{10},G_{11},G_{18},G_{19},G_{26},G_{27}\}$ $\{G_2,G_3,G_{10},G_{11},G_{18},G_{19},G_{26},G_{27}\}$	$\{G_{37},G_{64}\}$ Recycler Votes $\{G_2,G_3,G_{10},G_{11},G_{18},G_{19},G_{26},G_{27}\}$ $\{G_{64}\}$

	Fa	ursightedly Stable	: Case 1	
	i ₁	i ₂	i ₃	d
G ₇	$v_{i1}(G_7) = -2$	$v_{i2}(G_7) = -2$	$v_{i3}(G_7) = -2$	v_d (G ₇) = -6
G ₁₀	$v_{il}(G_{10}) = -2$	$v_{i2}(G_{10}) = -2$	$v_{i3}(G_{10}) = -2$	$v_d(G_{10}) = -6$
G ₁₉	$v_{il}(G_{19}) = -2$	$v_{i2}(G_{19}) = -2$	$v_{i3}(G_{19}) = -2$	$v_d(G_{19}) = -6$
G ₂₅	$v_{il}(G_{25}) = -2$	$v_{i2}(G_{25}) = -2$	$v_{i3}(G_{25}) = -2$	v_d (G ₂₅) = -6
G ₃₄	$v_{il}(G_{34}) = -2$	$v_{i2}(G_{34}) = -2$	$v_{i3}(G_{34}) = -2$	$v_d(G_{34}) = -6$
G ₃₇	$v_{il}(G_{37}) = -2$	$v_{i2}(G_{37}) = -2$	$v_{i3}(G_{37}) = -2$	$v_d(G_{37}) = -6$
G ₄₈	$v_{il}(G_{48}) = -2$	$v_{i2}(G_{48}) = -2$	$v_{i3}(G_{48}) = -2$	v_d (G ₄₈) = -6
G ₅₆	$v_{il}(G_{56}) = -2$	$v_{i2}(G_{56}) = -2$	$v_{i3}(G_{56}) = -2$	v_d (G ₅₆) = -6
G ₆₁	$v_{il}(G_{61}) = -2$	$v_{i2}(G_{61}) = -2$	$v_{i3}(G_{61}) = -2$	$v_d(G_{61}) = -6$
G ₆₄	$v_{il}(G_{64}) = -5/3$	$v_{i2}(G_{64}) = -5/3$	$v_{i3}(G_{64}) = -5/3$	v_d (G ₆₄) = -5
mean	-59/30	-59/30	-59/30	-5.9
median	-2	-2	-2	-6
mode	-2	-2	-2	-6

Table 4: Farsightedly Stable Payoffs, Simultaneous

Farsightedly Stable: Case 2

	i_1	i_2	i ₃	d
G ₃₇	$v_{il}(G_{37}) = -2$	$v_{i2}(G_{37}) = -2$	$v_{i3}(G_{37}) = -2$	$v_d(G_{37}) = -6$
G ₆₄	v_{il} (G ₆₄) = -5/3	v_{i2} (G ₆₄) = -5/3	$v_{i3}(G_{64}) = -5/3$	$v_d(G_{64}) = -5$
mean	-11/6	-11/6	-11/6	-5.5
median	-11/6	-11/6	-11/6	-5.5

		Nash Stable: Ca	ise 1	
	iı	i_2	i ₃	d
G_2	$v_{il}(G_2) = -4$	$v_{i2}(G_2) = -2$	$v_{i3}(G_2) = 0$	$v_d(\mathbf{G}_2) = -6$
G ₃	$v_{i1}(G_3) = -4$	$v_{i2}(G_3) = 0$	$v_{i3}(G_3) = -2$	$v_d(\mathbf{G}_3) = -6$
G ₁₀	$v_{il}(G_{10}) = -2$	$v_{i2}(G_{10}) = -2$	$v_{i3}(G_{10}) = -2$	$v_d(G_{10}) = -6$
G ₁₁	$v_{il}(G_{11}) = 0$	$v_{i2}(G_{11}) = -2$	$v_{i3}(G_{11}) = -4$	$v_d(G_{11}) = -6$
G ₁₈	$v_{il}(G_{18}) = -2$	$v_{i2}(G_{18}) = -4$	$v_{i3}(G_{18}) = 0$	$v_d(G_{18}) = -6$
G ₁₉	$v_{il}(G_{19}) = -2$	$v_{i2}(G_{19}) = -2$	$v_{i3}(G_{19}) = -2$	$v_d(G_{19}) = -6$
G ₂₆	$v_{il}(G_{26}) = 0$	$v_{i2}(G_{26}) = -4$	$v_{i3}(G_{26}) = -2$	$v_d(G_{26}) = -6$
G ₂₇	$v_{il}(G_{27}) = 0$	$v_{i2}(G_{27}) = -2$	$v_{i3}(G_{27}) = -4$	$v_d(G_{27}) = -6$
mean	-7/4	-9/4	-2	-6
median	-2	-2	-2	-6
mode	{0,2}	-2	-2	-6
		Nash Stable: Ca	use 2	
	iı	\mathbf{i}_2	i ₃	d
G_{64}	v_{il} (G ₆₄) = -5/3	$v_{i2}(G_{64}) = -5/3$	$v_{i3}(G_{64}) = -5/3$	$v_d(G_{64}) = -5$

		j ₃ (i ₃ ,i ₁)		
	$j_2(i_2,i_1)$	$j_2(i_2,i_2,)$	$j_2(i_2,i_3)$	$j_2(i_2,i_R)$
$j_1(i_1,i_1)$	(-6,0,0)	(-4,-2,0)	(-6,0,0)	(-4,-3,0)
$j_1(i_1,i_2)$	(-6,0,0)	(-2,-4,0)	(-6,0,0)	(-2,-4,0)
$j_1(i_1,i_3)$	(-6,0,0)	(-4,-2,0)	(-6,0,0)	(-4,-3,0)
$j_1(i_1,i_R)$	(-7,0,0)	(-5,-2,0)	(-7,0,0)	(-5,-1,0)
		j 3(i 3,i 2)		
	$j_2(i_2,i_1)$	$j_2(i_2,i_2,)$	$j_2(i_2,i_3)$	$j_2(i_2,i_R)$
$j_1(i_1,i_1)$	(-4,-2,0)	(-2,-4,0)	(-2,-4,0)	(-2,-5,0)
$j_1(i_1,i_2)$	(-4,-2,0)	(0,-6,0)	(0,-6,0)	(0,-7,0)
$j_1(i_1,i_3)$	(-2,-4,-0)	(0,-6,0)	(0,-6,0)	(0,-7,0)
$j_1(i_1,i_R)$	(-5,-2,0)	(-3,-4,0)	(-3,-4,0)	(-3,-3,0)
		j 3(i 3,i 3)		
	$j_2(i_2,i_1)$	$j_2(i_2,i_2,)$	$j_2(i_2,i_3)$	$j_2(i_2,i_R)$
$j_1(i_1,i_1)$	(-4,0,-2)	(-2,-2,-2)	(-2,0,-4)	(-2,-3,-2)
$j_1(i_1,i_2)$	(-4,0,-2)	(0,-4,-2)	(0,0,-6)	(0,-4,-2)
$j_1(i_1,i_3)$	(-2,0,-4)	(0,-2,-4)	(0,0,-6)	(0,-3,-4)
$j_1(i_1,i_R)$	(-5,0,-2)	(-3,-2,-2)	(-3,0,-4)	(-3,-1,-2)
		i.(i. i.)		
	i ₂ (i ₂ i ₁)	$j_3(i_3,i_R)$	ia(ia ia)	is(is is)
$\mathbf{i}_1(\mathbf{i}_1, \mathbf{i}_1)$	(-4.0-3)	(-2,-2,-3)	(-2.04)	(-2 -3 -1)
$i_1(i_1, i_2)$	(-4, 0, -3)	(2, 2, 3) (0, -4, -3)	(0.0-5)	(2, 3, 1) (0 - 4 - 1)
$i_1(i_1, i_2)$	(-2, 0, -4)	(0, -7, -4)	(0,0,-5)	(0, -3, -2)
JI(1,13)	(-2,0,-4)	(0, -2, -+)	(0,0,-3)	(0, -3, -2)

	Side Payments Not Per	nitted
	Recycler does not Vote	Recycler Votes
Unilateral	$\{[G_3,G_{19}],[G_9,G_{25}]\}$	$\{[G_3,G_{19}],[G_9,G_{25}]\}$
Majority	$\{[G_3,G_{19}],[G_9,G_{25}]\}$	$\{G_{37}\}$
~	$(\mathbf{C}_{\mathbf{r}})$	(C_{1})
Super Majority	{G ₃₇ }	{U ₃₇ }
Super Majority	{G ₃₇ } Side Payments Permi	{U ₃₇ }
Super Majority	{G ₃₇ } Side Payments Permi Recycler does not Vote	tted Recycler Votes
Super Majority	{G ₃₇ } <i>Side Payments Permi</i> Recycler does not Vote {G ₅₈ ,G ₅₉ ,G ₆₀ ,G ₆₂ ,G ₆₃ ,G ₆₄ }	{G ₃₇ } tted Recycler Votes {G ₅₈ ,G ₅₉ ,G ₆₀ ,G ₆₂ ,G ₆₃ ,G ₆₄ }
Unilateral Majority	{G ₃₇ } <i>Side Payments Permi</i> Recycler does not Vote {G ₅₈ ,G ₅₉ ,G ₆₀ ,G ₆₂ ,G ₆₃ ,G ₆₄ } {G ₅₈ ,G ₅₉ ,G ₆₀ ,G ₆₂ ,G ₆₃ ,G ₆₄ }	$\{G_{37}\}$ <i>tted</i> $Recycler Votes$ $\{G_{58},G_{59},G_{60},G_{62},G_{63},G_{64}\}$ $\{G_{58},G_{59},G_{60},G_{62},G_{63},G_{64}\}$

Side Payments Not Permitted			
	Recycler does not Vote	Recycler Votes	
Unilateral	$\{[G_3,G_{19}]\}$	$\{[G_3,G_{19}]\}$	
Majority	$\{[G_3,G_{19}]\}$	$\{G_{37}\}$	
C	$(\mathbf{C}_{\mathbf{r}})$	(C_{-})	
Super Majority	{ U ₃₇ }	{ U ₃₇ }	
Super Majority	{G ₃₇ } Side Payments Permi	{U ₃₇ }	
Super Majority	{G ₃₇ } Side Payments Permi Recycler does not Vote	tted Recycler Votes	
Unilateral	{G ₃₇ } Side Payments Permi Recycler does not Vote {G ₅₈ ,G ₅₉ ,G ₆₀ ,G ₆₂ ,G ₆₃ ,G ₆₄ }	$\{G_{37}\}$ <i>itted</i> Recycler Votes $\{G_{58},G_{59},G_{60},G_{62},G_{63},G_{64}\}$	
Unilateral Majority	{G ₃₇ } <i>Side Payments Permi</i> Recycler does not Vote {G ₅₈ ,G ₅₉ ,G ₆₀ ,G ₆₂ ,G ₆₃ ,G ₆₄ } {G ₅₈ ,G ₅₉ ,G ₆₀ ,G ₆₂ ,G ₆₃ ,G ₆₄ }	$\{G_{37}\}$ <i>itted</i> $Recycler Votes$ $\{G_{58},G_{59},G_{60},G_{62},G_{63},G_{64}\}$ $\{G_{58},G_{59},G_{60},G_{62},G_{63},G_{64}\}$	

Table 9: Farsightedly and Nash Stable Payoffs, Sequential

Case 1				
	i ₁	i ₂	i ₃	d
[G ₃ ,G ₁₉]	$v_{il}([G_3,G_{19}]) = -4$	$v_{i2}([G_3,G_{19}]) = -2$	$v_{i3}([G_7,G_{19}]) = 0$	$v_d([G_3,G_{19}]) = -6$
$[G_9, G_{25}]^*$	$v_{i1}([G_9,G_{25}]) = -4$	$v_{il}([G_9,G_{25}]) = -2$	$v_{il}([G_9,G_{25}]) = 0$	$v_{il}([G_9,G_{25}]) = -6$

	Case 2			
	i ₁	i ₂	i ₃	d
G ₃₇	$v_{il}(G_{37}) = -2$	$v_{i2}(G_{37}) = -2$	$v_{i3}(G_{37}) = -2$	v_d (G ₃₇) = -6

Case	3

	i ₁	i_2	i ₃	d
G ₅₈	$v_{il}(G_{58}) = -3.5$	$v_{i2}(G_{58}) = -1.5$	$v_{i3}(G_{58}) = 0$	v_d (G ₅₈) = -5
G59	$v_{il}(G_{59}) = -3.5$	$v_{i2}(G_{59}) = -1.5$	$v_{i3}(G_{59}) = 0$	v_d (G ₅₉) = -5
G ₆₀	$v_{il}(G_{60}) = -3.5$	$v_{i2}(G_{60}) = -1.5$	$v_{i3}(G_{60}) = 0$	v_d (G ₆₀) = -5
G ₆₂	$v_{il}(G_{62}) = -3.5$	$v_{i2}(G_{62}) = -1.5$	$v_{i3}(G_{62}) = 0$	v_d (G ₆₂) = -5
G ₆₃	$v_{il}(G_{63}) = -3.5$	$v_{i2}(G_{63}) = -1.5$	$v_{i3}(G_{63}) = 0$	v_d (G ₆₃) = -5
G ₆₄	v_{il} (G ₆₄) = -3.5	$v_{i2}(G_{64}) = -1.5$	$v_{i3}(G_{64}) = 0$	v_d (G ₆₄) = -5

Case 4

	i ₁	i ₂	i ₃	d
G ₅₈	$v_{il}(G_{58}) = -5/3$	$v_{i2}(G_{58}) = -5/3$	$v_{i3}(G_{58}) = -5/3$	v_d (G ₅₈) = -5
G ₅₉	$v_{il}(G_{59}) = -5/3$	$v_{i2}(G_{59}) = -5/3$	$v_{i3}(G_{59}) = -5/3$	v_d (G ₅₉) = -5
G ₆₀	$v_{il}(G_{60}) = -5/3$	$v_{i2}(G_{60}) = -5/3$	$v_{i3}(G_{60}) = -5/3$	v_d (G ₆₀) = -5
G ₆₂	$v_{i1}(G_{62}) = -5/3$	$v_{i2}(G_{62}) = -5/3$	$v_{i3}(G_{62}) = -5/3$	v_d (G ₆₂) = -5
G ₆₃	$v_{il}(G_{63}) = -5/3$	$v_{i2}(G_{63}) = -5/3$	$v_{i3}(G_{63}) = -5/3$	v_d (G ₆₃) = -5
G ₆₄	$v_{il}(G_{64}) = -5/3$	$v_{i2}(G_{64}) = -5/3$	$v_{i3}(G_{64}) = -5/3$	v_d (G ₆₄) = -5

*This is only Farsightedly Stable, not Nash.





j₃⊖

G₄₈









Case 2

FIGURE 3: NASH STABLE NETWORKS, SIMULTANEOUS

Case 1





FIGURE 4: COLLECTIVE-CHOICE LEVEL SUPERNETWORKS, SIMULTANEOUS

-VVII



ŃŴ ίv

Unilateral

Majoritarian

Super Majoritarian

Figure 5: Farsightedly and Nash Stable Networks, Sequential





Note: Networks G9 and G25 are only farsightedly stable, not Nash stable.

Case 2







G₆₀











G₆₄

Case 4

j



Ð

Ĵ2

æ

G 58

Js Ĵr



Ð

Ĵ2

G 59

Ð

ji 🚯

j'ı

Ĵr

j'ı



Ĵ2

G 58

Ð

j2

j,

j'n

Ĵr

j"ı

Js

Case 3



Figure 6: Collective-Choice Level Supernetworks, Sequential



Figure 7: Constitutional Level Supernetworks



Constitutional

Figure 8: The Nesting Property



- Agranoff, R. and McGuire, M. 2001. "Big Questions in Public Network Management Research." *Journal of Public Administration Research and Theory*, 11(3), 295-326.
- Bala, V. and Goyal, S. 2000. "A Noncooperative Model of Network Formation." *Econometrica*, 68(5), 1181-1229.
- **Chwe, M.** 2000. "Communication and Coordination in Social Networks." *Review of Economic Studies*, 67(1), 1-16.
- Coase, R. H. 1937. "The Nature of the Firm." *Economica*, 4(16), 386-405.
- Dutta, B. and Mutuswami, S. 1997. "Stable Networks." Journal of Economic Theory, 76(2), 322-344.
- Hardin, G. 1968. "The Tragedy of the Commons." Science, 162(3859), 1243-1248.
- Jackson, M. O. and Wolinsky, A. 1996. "A Strategic Model of Social and Economic Networks." *Journal of Economic Theory*, 71, 44-74.
- McGinnis, M. D. 2011. "An Introduction to Iad and the Language of the Ostrom Workshop: A Simple Guide to a Complex Framework." *Policy Studies Journal*, 39(1), 169-183.
- Myerson, R. B. 1977. "Graphs and Cooperation in Games." *Mathematics of Operations Research*, 2(3), 225-229.
- North, D. C. 1991. "Institutions." Journal of Economic Perspectives, 5(1), 97-112.
- **Ostrom, E.** 2009. "A General Framework for Analyzing Sustainability of Social-Ecological Systems." *Science*, 325(5939), 419-422.
 - ___. 2007. "Institutional Rational Choice: An Assessment of the Institutional Analysis and Development Framework," P. A. Sabatier, *Theories of the Policy Process*. Boulder, CO: Westview Press, 21-64.
 - _. 2005. Understanding Institutional Diversity. Princeton: Princeton University Press.
- Page, J., Frank H. and Wooders, M. H. 2007a. "Club Networks with Multiple Memberships and Noncooperative Stability," *Conference in Honor of Ehud Kalai.*
 - _. 2007b. "Network Formation Games." Indiana University Working Paper.
- Page, J., Frank H.; Wooders, M. H. and Kamat, S. 2005. "Networks and Farsighted Stability." *Journal of Economic Theory*, 120(2), 257-269.
- ____. 2002. "Networks and Farsighted Stability." University of Warwick Working Paper No 621.
- Shapley, L. S. and Shubik, M. 1969. "On the Core of an Economic System with Externalities." *The American Economic Review*, 59(4), 678-684.
- Simon, H. A. 1955. "A Behavioral Model of Rational Choice." *Quarterly Journal of Economics*, 69(1), 99-118.
- Williamson, O. E. 1975. *Markets and Hierarchies: Analysis and Antitrust Implications*. New York, NY: Free Press.